CAREER POINT

Study Material for Pre foundation Class 9

Prepared by Career Point Kota Experts

CONTENTS OF THE PACKAGE AT A GLANCE

Class IX

Physics

- Describing Motion Around Us
- How Forces Affect Motion
- Work, Energy And Simple Machines
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- Gravitation
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- NCERT has released Class 9 Mathematics Set-1 textbook, while Set-2 has not been released yet

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- Classification

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- NCERT Class 9 textbook Set-1 has not been released yet

Social Science [Set-2]

- NCERT Class 9 textbook Set-2 has not been released yet

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- The Lost Child (Moments Fiction)
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- No Men Are Foreign (Beehive Poetry)
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- Weathering The Storm in Ersama (Moments Fiction)
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- Pronoun
- Adjective
- Adverb
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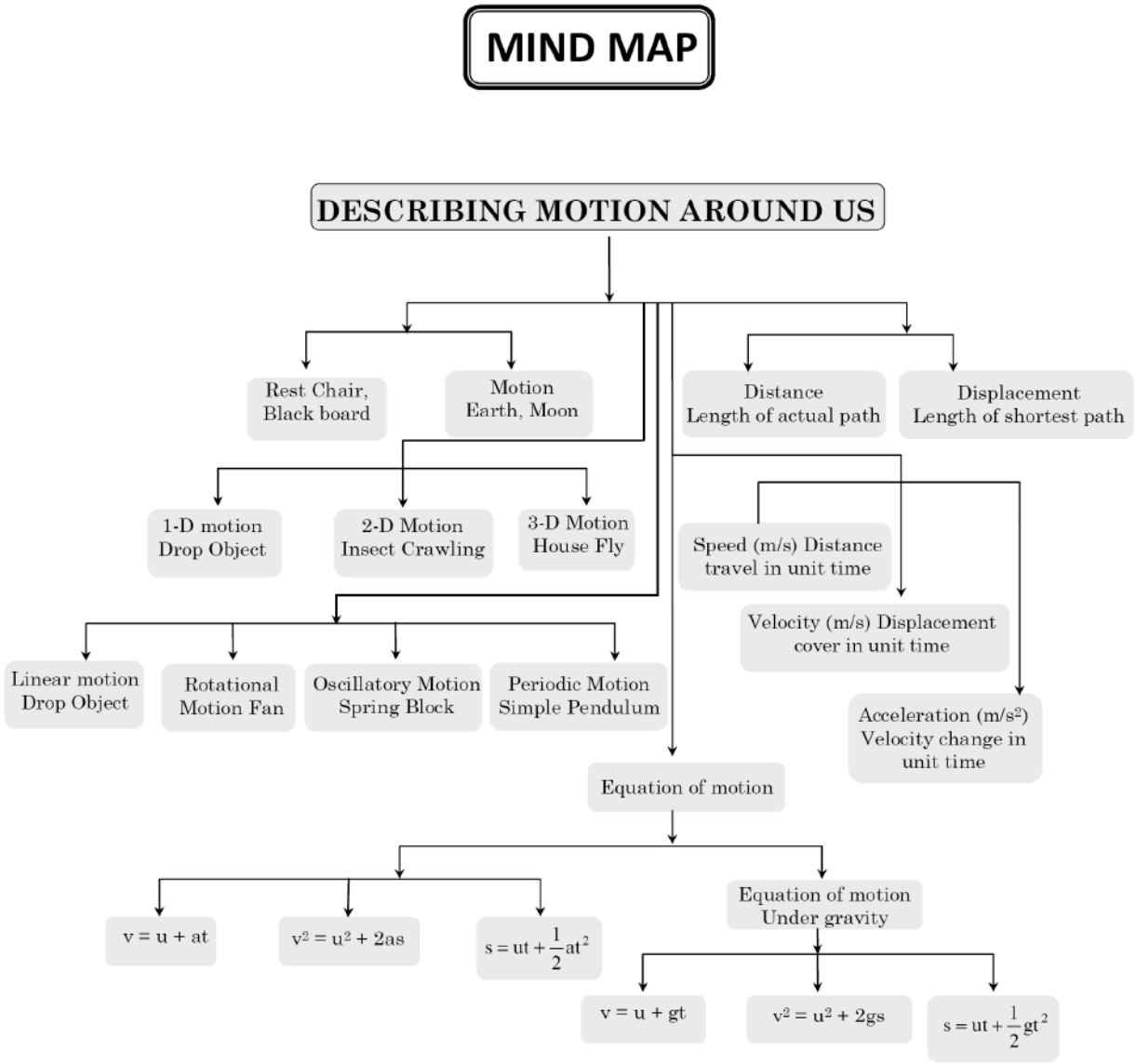
- Prepositions
- Tense
- Passivization
- Reported speech
- Modals
- Subject verb concord
- Conditionals
- Error spotting
- Comprehension
- Fillers
- Cloze test
- Synonyms & antonyms
- Idioms and phrases
- Phrasal verbs
- One word substitution
- Spelling

Note to the Students

Career Point offers this Class 9 Study Package to support complete learning for school curriculum as well as Olympiad and foundation-level exams. This sample represents our set of nine books: **Physics**, **Chemistr**y, **Biology**, two **Mathematics** books, two **Social Science** books, two **English** books and **Mental Ability**. Each book provides clear concepts, illustrative examples and ample practice to strengthen understanding and build analytical skills. The material is designed to develop a strong academic base and prepare students with confidence for higher classes and future competitive studies.

Mind Map

Each chapter contains many articles (Concepts, Theories etc.). Mind map interconnect all these articles logically. By this student can understand whole chapter articles interconnectivity clearly in a single picture frame.



Theory & Concepts

Each chapter consist of exhaustive theory which gives conceptual clarity and command over topics. Appropriate explanation of theory with the help of images, diagrams, flowcharts, mind maps, info graphics, and tables.

DESCRIBING MOTION AROUND US

Physics

It is the branch of science in which we observe, measure and describe nature and natural phenomena.

◈ **Mechanics**

It is the branch of physics which deals with the study of objects in the condition of rest or motion. It is divided into two parts

(i) Statics (ii) Kinematics and Dynamics

(i) Statics: It deals with the study of objects at rest or in equilibrium, even when they are under the action of several forces.

(ii) Kinematics and Dynamics:

Kinematics: It deals with the study of motion of objects without considering the cause of motion.

e.g.: equations of motion

Competitive Level

Competitive level is specially designed for competition exam rquirements and to better understanding the concepts, well explained theory, clearly explained formulas with good number of quality examples are given in this.

COMPETITIVE LEVEL

- **Representation of a vector:** A vector is represented by a directed line segment drawn in the given direction on a certain scale.The length of line shows its magnitude and arrow shows direction.

Tail ⟶ head

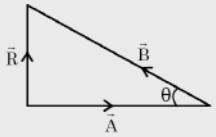
(symbolic representation)

In Chapter Example

To understand the application of concepts, there is ***in chapter solved example*** are given. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

Ex.5 The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

Sol. Since $\vec{R}$ is perpendicular to $\vec{A}$. Figure shows the three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$.



angle between $\vec{A}$ and $\vec{B}$ is $\pi - \theta$

$\sin\theta = \frac{R}{B} = \frac{R}{2B} = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

$\Rightarrow$ angle between A and B is 150°.

Practice Exercises

Includes three sets of exercises covering all the topics. Helps the students to assess their strengths and weaknesses and work on them accordingly. Separate exercises for subjective as well as objective questions and previous year competitive exams questions (NTSE, Olympiads)

EXERCISE-1

➢ Very Short Answer Type Questions

Q.1 A ball is thrown up with a certain velocity. It attains a height of 40 m and comes back to the thrower. Find the distance and magnitude of displacement.

Q.2 What is the S.I. unit of displacement?

Q.3 A horse runs a distance of 1200 m in 3 min and 20 s. What is the speed of the horse?

Q.4 What is the S.I. unit of velocity?

Q.5 What is the S.I. unit of acceleration?

➢ Short Answer Type Questions – Type I

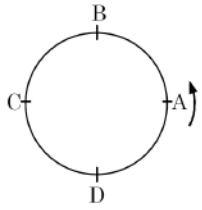
Q.6 Distance and displacement are equal in some cases. Give reasons.

➢ Short Answer Type Questions – Type II

Q.11 Give three examples to explain that motion is relative.

Q.12 Which of the following is true for displacement?
(i) It cannot be zero.
(ii) Its magnitude is greater than the distance travelled by the object.

Q.13 A particle moves along a circle of radius R as shown in figure. It starts from A and moves in anti-clockwise direction.



Calculate the distance travelled and displacement:

EXERCISE-2

Q.1 ABC is the shortest path length between the two points and ADC is the actual path length. Then which of the two corresponds to displacement?
(A) ADC (B) ABC
(C) Can't say (D) None of these

Q.2 Rest and motion both are
(A) Relative terms
(B) Absolute terms
(C) Can't say
(D) None of these

Q.3 An object has travelled 10 km in 15 minutes, its displacement will be
(A) 10 km
(B) zero
(C) More than 10 km
(D) Cannot be predicted

Q.4 Which of the following does not need direction to be defined completely

Q.8 From the top of a tower, a particle is projected upwards and it reaches the ground after 5 s. The initial velocity of the particle is 12 m/s, the height of the tower is
(A) 55 m (B) 65 m
(C) 75 m (D) 85 m

Q.9 A train passes over a 400 m long bridge. If the speed of the train is 30 m/s and the train takes 20 s to cross the bridge, then the length of the train is
(A) 400 m (B) 600 m
(C) 800 m (D) 200 m

Q.10 If a body covers a distance d with velocity v_1 and another distance d with same velocity v_2, then average velocity for the whole journey would be equal to
(A) $\frac{2v_1v_2}{v_1+v_2}$ (B) $\frac{v_1v_2}{v_1+v_2}$
(C) $\frac{v_1v_2}{2v_1+v_2}$ (D) $\frac{2(v_1v_2)}{v_1v_2}$

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

Q.1 Value of one Fermi is

(A) 10^{-13}metre

(B) 10^{-14} metre

(C) 10^{-15} metre

(D) 10^{-16} metre

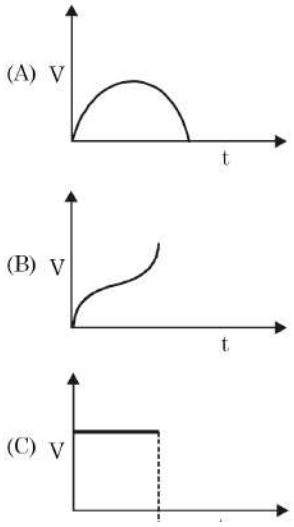
Q.2 A student starts with a velocity 40 km/hr for school at 4 km away from his house. Due to closing of school he returns soon to his house with a velocity of 60km/hr.His average velocity will be

(A) zero

(B) 10 km/hr

(C) 48 km/hr

(D) 50 km/hr



Answer key

Answer key is provided at the end of the exercise sheets.

ANSWER KEY

EXERCISE - 1

1. 80 m and zero

3. 6m/s

7. 2h, 0

10. (i) 100 m, 100 m (ii) 112.5 m, 87.5 m

13. (i) $\frac{\pi R}{2}, \sqrt{2}R$ (ii) $\pi R, 2R$ (iii) $\frac{3\pi R}{2}, \sqrt{2}R$

14. 40km/hr, 666.67m/min, 11.11m/sec

15. 3.069 km/s

20. 8 m/sec^2, 5.5s

24. 22.2 m/s

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	D	A	C	B	A	B	D	A	B	C	D	C	B
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	D	C	A	B	D	B	A	A	A	A	B	B	C

EXERCISE - 3

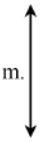
Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	B	D	D	B	B	D	A	C	D	A	B	B	B
Ques.	16	17	18	19	20										
Ans.	B	C	A	A	B										

SOLUTIONS

EXERCISE-1

➢ Very Short Answer Type Questions

Sol.1 40 m.



Displacement = 0
Distance = 40m + 40m = 80m

Sol.2 Metre

Sol.3 $\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{1200}{200} = 6 \text{ m/sec.}$

Sol.4 m/sec

Sol.5 m/sec^2

(i) Distance = Displacement = Area A_1
= 100 m.

(ii) Displacement = Area A_1 + Area A_2
= 100 – 12.5 = 87.5 m

Distance = (Area A_1) + (Area A_2)
= 100 + 12.5 = 112.5 m.

➢ Short Answer Type Questions – Type II

Sol.11 A train is moving on the track, the passenger are seated, will be stationary w.r.t. to each other but in moving condition w.r.t. to station.

Sol.12 Both are incorrect.

Sol.13 (i) $A \rightarrow B$

$$\text{Distance} = \frac{1}{4}(2\pi R) = \frac{\pi}{2}R$$

PHYSICS

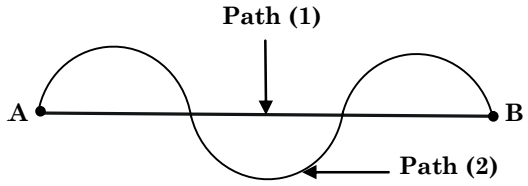
Chapter

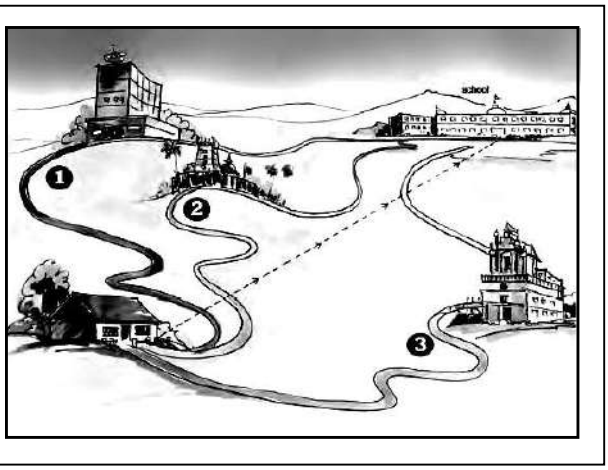
1

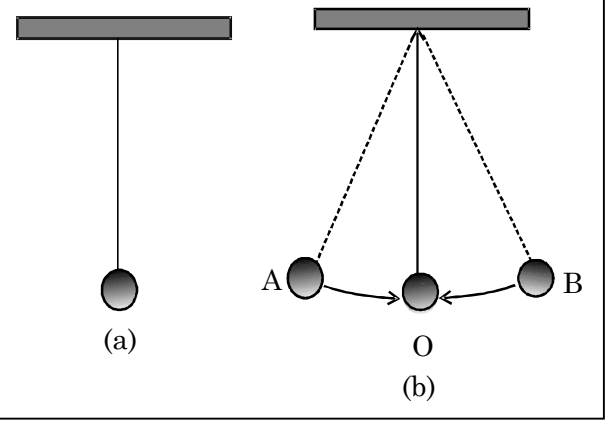
Describing Motion Around Us

Chapter Outline

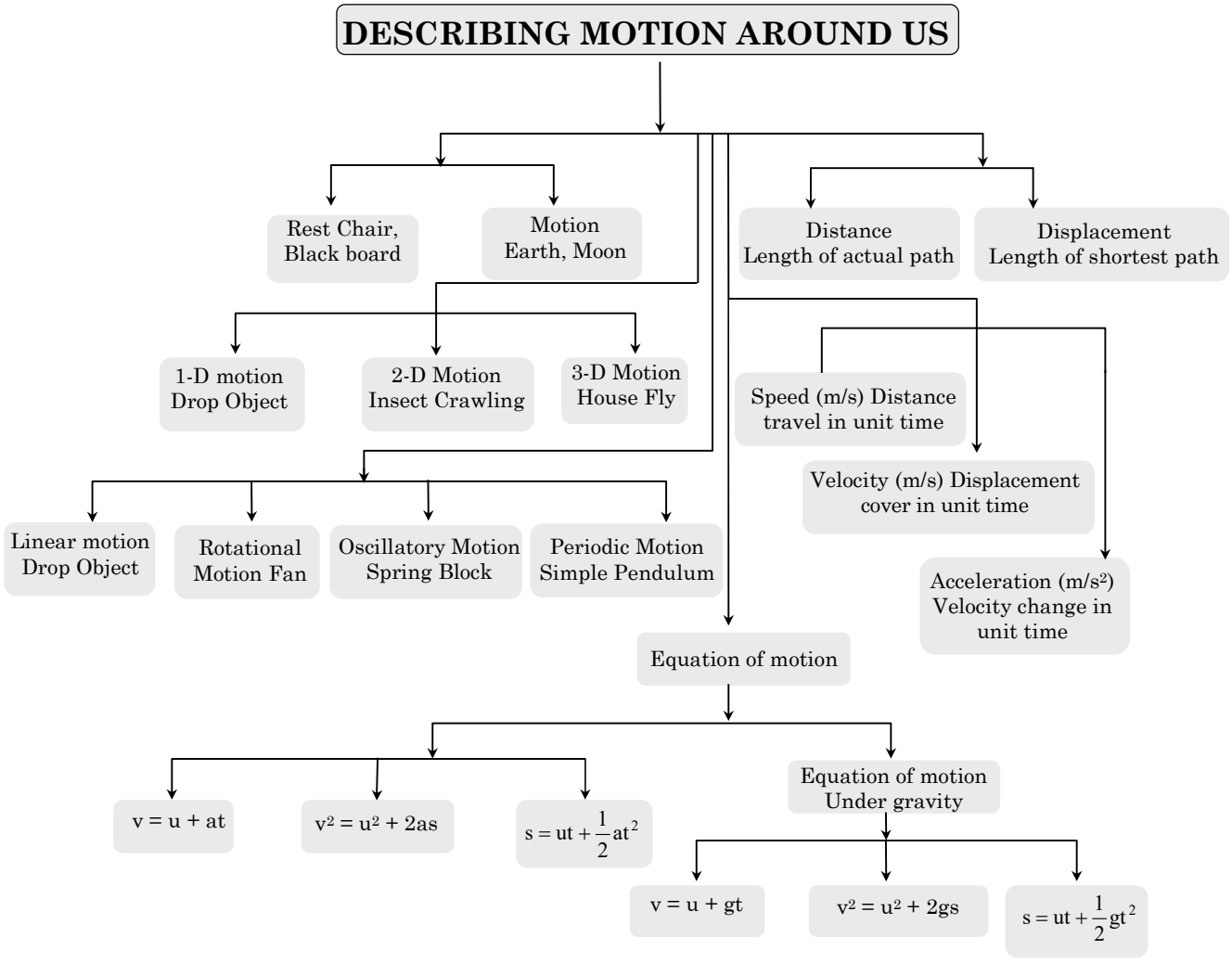
- Physics
- Rest and Motion
- Types of motion
- Scalar and Vector Quantity
- Distance and displacement
- Speed & Velocity
- Uniform and non uniform motion
- Acceleration
- Graphical representation of motion
- Equation of uniformly accelerated motion
- Circular motion







MIND MAP


DESCRIBING MOTION AROUND US
Rest Chair, Black board
Motion Earth, Moon
Distance Length of actual path
Displacement Length of shortest path
1-D motion Drop Object
2-D Motion Insect Crawling
3-D Motion House Fly
Speed (m/s) Distance travel in unit time
Velocity (m/s) Displacement cover in unit time
Acceleration (m/s²) Velocity change in unit time
Linear motion Drop Object
Rotational Motion Fan
Oscillatory Motion Spring Block
Periodic Motion Simple Pendulum
Equation of motion
v = u + at
v² = u² + 2as
s = ut + 1/2 at²
Equation of motion Under gravity
v = u + gt
v² = u² + 2gs
s = ut + 1/2 gt²

DESCRIBING MOTION AROUND US

Physics

It is the branch of science in which we observe, measure and describe nature and natural phenomena.

◈ Mechanics

It is the branch of physics which deals with the study of objects in the condition of rest or motion. It is divided into two parts

(i) Statics (ii) Kinematics and Dynamics

(i) Statics: It deals with the study of objects at rest or in equilibrium, even when they are under the action of several forces.

(ii) Kinematics and Dynamics:

Kinematics: It deals with the study of motion of objects without considering the cause of motion.

e.g.: equations of motion

Dynamics: It deals with the study of motion of objects with considering the cause of motion.

e.g.: Newton's laws of motion

Rest & Motion

◈ Rest

An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time.

e.g.: The chair, black board, table in the class room are at rest w.r.t. the students.

◈ Motion

A body is said to be in motion if its position changes continuously w.r.t. the surroundings (or with respect to an observer) with the passage of time.

e.g.: A car moving on the road will be in motion w.r.t. to the person standing on the road

Rest and motion are relative terms, there is nothing like absolute motion or rest.

e.g.: A train is moving on the track, the passengers are seated, will be stationary with respect to each other but in moving condition with respect to station.

Therefore, all the motions are relative. There is nothing like absolute motion.

To study the motion of an object, following points are essential:

◈ Concept of a Point Object

In mechanics while studying the motion of an object, sometimes its dimensions are not important and the object may be treated as a point object without much error. When the size of the object is much less in comparison to the distance covered by the object then the object is considered as a point object.

e.g.: Earth can be considered as a point object for studying its motion around the sun. Because length of the path covered by the earth in one revolution is very large in comparision to the size of earth, so that earth can be considered as a point object.

◈ Frame of Reference

A fixed point or a fixed object with respect to which the given body changes its position is known as reference point or origin

To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axis and name them x-y-z axis. The co-ordinates (x, y, z) of the particle then specify the position of object w.r.t. that frame. If any one or more co-ordinates change with time, then we say that the object is moving w.r.t. this frame.

Note: *We need Frame of reference to define dimensions of motion.*

Types of Motion

◈ Motion in 1-D

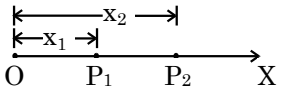
If only one of the three co-ordinates specifying the position of object changes w.r.t. time then its motion is known as 1D motion. In such a case the object moves along a straight line. This motion is also known as rectilinear or linear motion.

e.g.:

(i) Motion of train along straight railway track.

(ii) An object falling freely under gravity.

(iii) When a particle moves from P_1 to P_2 along a straight line path only the x-co-ordinate changes.



◈ Motion in 2-D

If two of the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called two dimensional. In such a motion the object moves in a plane.

e.g:

(i) Motion of queen on carom board.

(ii) An insect crawling on the floor of the room.

(iii) Motion of object in horizontal and vertical circles etc.

(iv) Motion of planets around the sun.

(v) A car moving along a zig-zag path on a level road.

◈ Motion in 3-D

If all the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called 3-D. In such a motion the object moves in a space.

e.g.:

(i) A bird or kite flying in the sky (Also kite).

(ii) Random motion of gas molecules.

(iii) Motion of an aeroplane in space.

◈ Linear Motion (or Translatory Motion)

The straight line motion is called linear motion.

e.g.: The motion of a car moving on straight road, a running person, a stone being dropped, motion of a train on a straight track

◈ Rotational Motion

Motion of a body around a fixed axis is called rotational motion.

e.g.: The motion of an electric fan, motion of earth about its own axis.

◈ Oscillatory Motion

The to and fro periodic motion of a body around a fixed point is called oscillatory motion.

e.g.: The motion of a simple pendulum, a body suspended from a spring.

Scalar and Vector Quantity

Physical quantities (i.e. quantities of physics) can be divided into two types:

◈ Scalar quantity

Any physical quantity, which can be completely specified by its magnitude, is known as scalar quantity or a scalar.

e.g.: Charge, distance, area, speed, time, temperature, density, volume, work, power, energy, pressure, potential etc.

◈ Vector quantity

The quantity which can be determined by its magnitude and direction and also can be added or subtracted by vector algebra, is called a vector quantity.

e.g.: Displacement, velocity, acceleration, force, momentum, weight, electric field etc.

◈ Difference between scalar and vector

Scalar	Vector
1. They have magnitude only.	1. They have magnitude as well as direction.
2. They are added or subtracted arithmetically like 3 kg + 5 kg = 8 kg	2. They are added or subtracted by the process of vector addition.

COMPETITIVE LEVEL

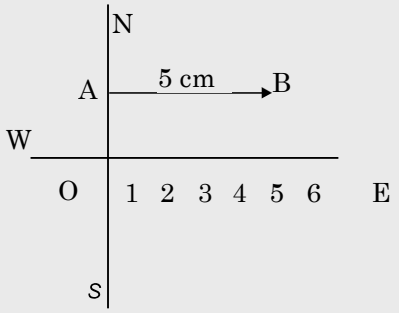
- **Representation of a vector:** A vector is represented by a directed line segment drawn in the given direction on a certain scale.The length of line shows its magnitude and arrow shows direction.

Tail ⟶ head

(symbolic representation)

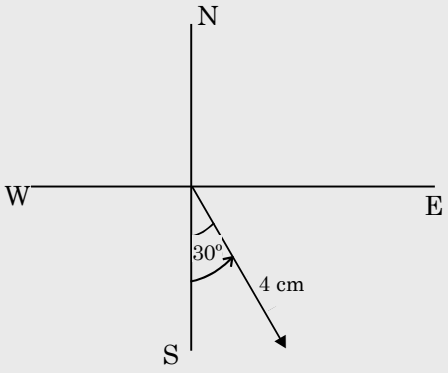
Ex.1 Represent a displacement of 50 m towards east.

Sol. Take the scale 10 m = 1 cm



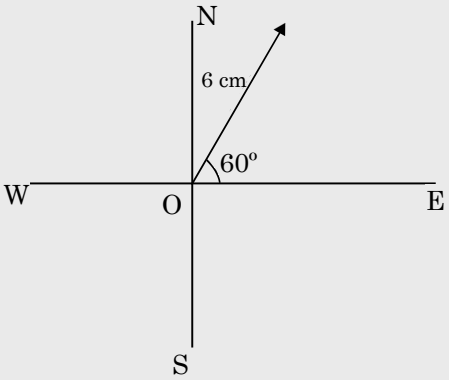
Ex.2 Represent a velocity of 20 km/h towards 30º east of south.

Sol. Take scale 5 km/h = 1 cm.



Ex.3 Represent on graph 6 m displacement, 60º north–east (north of east)

Sol. Take scale 1 m = 1 cm



◈ Addition of Vectors

Two or more vectors are added by following laws:

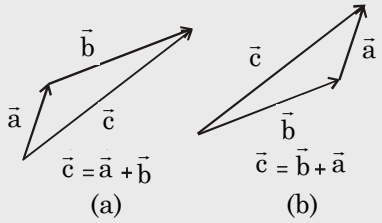
(i) triangle law

(ii) parallelogram law

(iii) polygon law

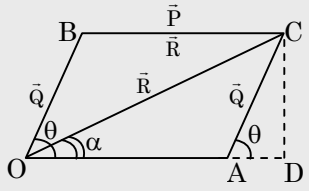
- **Triangle law of vector addition:** If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these two vectors is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

  Given two vectors $\vec{a}$ and $\vec{b}$, put the tail of $\vec{b}$ at the head of $\vec{a}$, then the sum of $\vec{a}$ and $\vec{b}$ is defined as the vector $\vec{c}$ drawn from the tail of $\vec{a}$ to the head of $\vec{b}$.



- **Parallelogram Law of Vector Addition Statement:** If two vectors acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through that point represents the resultant in magnitude and direction.

- **Analytical approach to parallelogram law of vector addition:** Let the two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ of the parallelogram OACB. Suppose the angle between the vectors is θ, i.e. $\angle AOB = \theta$. According to parallelogram law of vector addition, the diagonal represents the resultant $\vec{R}(\overrightarrow{OC})$ in magnitude and direction. Suppose $\vec{R}$ makes an angle α with $\vec{P}$ i.e. $\angle AOC = \alpha$



Magnitude of Resultant:

$\therefore \quad R = \sqrt{P^2 + q^2 + 2PQ\cos\theta}$

Direction of resultant:

$\therefore \quad \tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

Ex.4 Find the resultant of two forces each having magnitude F_0, and angle between them is θ.

Sol. $F^2_{Resultant} = F_0^2 + F_0^2 + 2F_0^2\cos\theta$

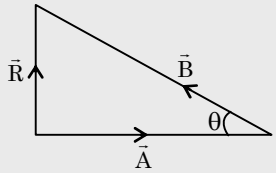
$= 2F_0^2(1 + \cos\theta) = 2F_0^2 \times 2\cos^2\dfrac{\theta}{2}$

$2 \times 2\cos^2\dfrac{\theta}{2}$

$F_{resultant} = 2F_0\cos\dfrac{\theta}{2}$

Ex.5 The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

Sol. Since $\vec{R}$ is perpendicular to $\vec{A}$. Figure shows the three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$.



angle between $\vec{A}$ and $\vec{B}$ is $\pi - \theta$

$\sin\theta = \frac{R}{B} = \frac{R}{2B} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30^o$

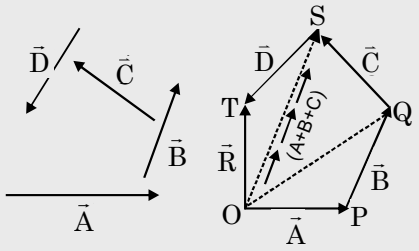
$\Rightarrow$ angle between A and B is 150^o.

◈ Polygon Law of Vector Addition

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the resultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.

Let the number of vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ etc. be acting in different directions as shown in figure.

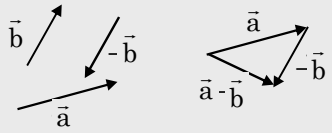
To find their resultant vector, coincide the tail of $\vec{B}$ with the head of $\vec{A}$, tail of $\vec{C}$ with the head of $\vec{B}$ and tail $\vec{D}$ with the head of $\vec{C}$. Then the single vector drawn form the tail of $\vec{A}$ to head of $\vec{D}$.



will thus, it is clear it the vectors $\vec{A}$ $\vec{B}$ $\vec{C}$ and $\vec{D}$ are represented in magnitude and direction by the sides $\overrightarrow{OP}$, $\overrightarrow{PQ}$, $\overrightarrow{QS}$ and $\overrightarrow{ST}$ of an open polygon taken in the same order, then their resultant vector $\vec{R}$ will be represented in magnitude and direction by the closing side $\overrightarrow{OT}$ of the polygon taken in opposite order. This method of finding the resultant is called polygon law of vectors.

- **Subtraction of Vectors:** The negative of a vector is defined as a vector of same magnitude but opposite direction.

The Subtraction of a vector $\vec{b}$ from another vector $\vec{a}$ is defined as the addition of $-\vec{b}$ to $\vec{a}$, as shown in figure

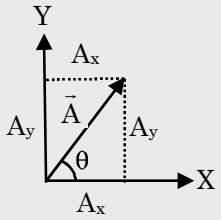


- **Resolution of A Vector:** It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is splited are called component vectors. Infect, the resolution of a vector is just opposite to composition of vectors.

  A vector can have infinite component vectors but for simplicity a vector is resolved into two or three mutually perpendicular components

- **Resolution into mutually Perpendicular Vectors in a plane (2D Resolution):** The figure shows resolution of a vector $\vec{A}$ into two oblique vectors $\vec{A}_x$ and $\vec{A}_y$.

  Using the elementary knowledge of trigonometry the vectors $\vec{A}_x$ and $\vec{A}_y$ are given by $\vec{A}_x = \vec{A}\cos\theta$, $\vec{A}_y = \vec{A}\sin\theta$



Resolution of a vector into mutually perpendicular components

◈ Scalar Product

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}.\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of their magnitude with cosine of angle

between them. Thus, $\vec{A}.\vec{B} = AB\cos\theta$ {here θ is the angle between the vectors}

If the scalar product of two nonzero vectors vanishes then the vectors are perpendicular.

The scalar product of a vector by itself is termed as self dot product and is given by

$(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = AA\cos0^o = A^2$

$\Rightarrow \quad A = \sqrt{\vec{A}.\vec{A}}$

In case of unit vector $\hat{n}$,

$\hat{n}.\hat{n} = 1 \times 1 \times \cos 0^o = 1$

$\Rightarrow \hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$;

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = [A_xB_x + A_yB_y + A_zB_z]$

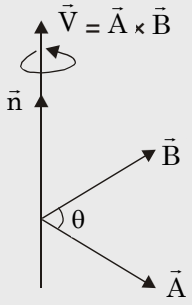
◈ Vector Product

The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A} \times \vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as:

$$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$$

Here θ is the angle between the vectors and the direction $\hat{n}$ is given by the right-hand-thumb rule.

- **Right-Hand-Thumb Rule:** To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ and $\vec{B}$ with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of.



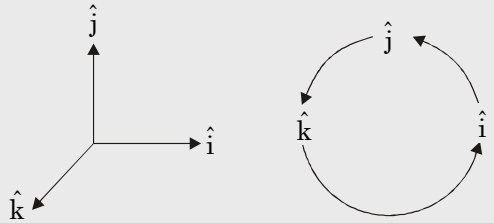
$\vec{A} \times \vec{A} = AA \sin 0^o \hat{n} = \vec{0}$.

In case of unit vector $\hat{n}$, $\hat{n} \times \hat{n} = \vec{0}$

$\Rightarrow \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

In case of orthogonal unit vectors $\hat{i}, \hat{j}, \hat{k}$

in accordance with right-hand-thumb-rule, $\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{i} = \hat{j}$



In terms of components, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$= i\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ Bx & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$

$\vec{A} \times \vec{B} = \hat{i}(A_yB_z - A_z B_y) - \hat{j}(A_xB_z - A_z B_x) + \hat{k}(A_xB_y - A_y B_x)$

The magnitude of area of the parallelogram formed by the adjacent sides of vector and equal to $|\vec{A} \times \vec{B}|$.

Ex.6 If the Vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. Find the value of a?

Sol. If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$\Rightarrow \vec{P} \cdot \vec{Q} = 0$

$\Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}) \,.\, (a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$\Rightarrow a^2 - 2a - 3 = 0$

$\Rightarrow a^2 - 3a + a - 3 = 0$

$\Rightarrow a(a - 3) + 1(a - 3) = 0$

$\Rightarrow a = -1, 3$

Ex.7 Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Find a unit vector which is perpendicular to both $\vec{A}$ and $\vec{B}$.

Sol. $\vec{A} \times \vec{B} = AB \sin\theta \quad \Rightarrow \quad \hat{n} = \dfrac{\vec{A} \times \vec{B}}{AB\sin\theta}$

here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Ex.8 Find $\vec{A} \times \vec{B}$, if $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$

Sol. $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix}$$

$= \hat{i}\,(-4 - (-4)) - \hat{j}(2 - 12) + \hat{k}\,(-1 - (-6)) = 10\hat{j} + 5\hat{k}$

Distance and Displacement

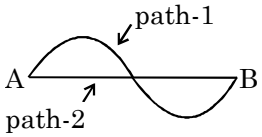
◈ **Distance:**

It is the actual length of path covered by a moving particle . It is a scalar quantity. Its S.I. unit is metre (m).

◈ **Displacement:**

It is the shortest distance between the initial and final position of the particle. It is a vector quantity. Its S.I. unit is metre (m).

e.g.: Consider a body moving from a point A to a point B along the path shown in figure. Then total length of path covered is called distance (path-1). While the length of straight line AB in the direction from A to B is called displacement (path-2).



Note : *If a body travels in such a way that it comes back to its starting position, then the displacement is zero. However, distance travelled is never zero in case of moving body.*

◈ **Some important points:**

(i) When an object moves towards right from origin, its displacement consider as positive.

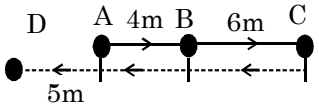
(ii) When an object moves towards left from origin its displacement consider as negative.

(iii) When an object remains stationary or it moves first towards right and then an equal distance towards left, its displacement is zero.

(iv) Shifting origin causes no change in displacement.

(v) If body moves along the circumference of the circle of radius r then distance travelled by it is given by $2\pi r$ and displacement is given by zero, for one complete revolution.

Ex.9 A body starts from A and moves according to given figure. (Body retraces the path after C then reaches to D)



The distance and displacement are as follows for different path

Sol.

Path	Distance	Displacement
AB	4m	4m
ABC	10m	10m
ABCB	16m	4m
ABCA	20m	0m
ABCAD	25m	–5m

◈ **Difference between distance and displacement**

Distance	Displacement
1. Distance is the length of the path actually traveled by a body in any direction.	1. Displacement is the shortest distance between the initial and the final positions of a body in the direction of the point of the final position.
2. Distance between two given points depends upon the path chosen.	2. Displacement between two points is measured by the straight path between the points.
3. Distance is always positive.	3. Displacement may be positive as well as negative and even zero.
4. Distance is a scalar quantity.	4. Displacement is a vector quantity.
5. Distance will never decrease.	5. Displacement may decrease.

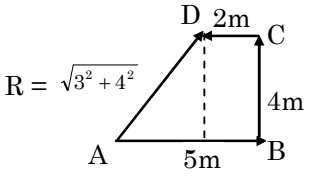
Ex.10 A person travels a distance of 5 m towards east, then 4 m towards north and then 2 m towards west.

(i) Calculate the total distance travelled.

(ii) Calculate the resultant displacement.

Sol. (i) Total distance travelled by the person = 5 m + 4 m + 2 m = 11 m

(ii) The resultant displacement is calculated by joining the initial position A to the final position D. Hence, the displacement of the person = 5m towards AD.

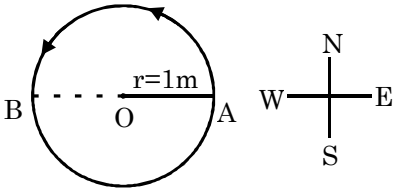


Ex.11 A person moves in a circular path centered at O. He starts from A and reaches diametrically opposite point B. Then find:

(i) distance between A and B

(ii) displacement between A and B

Sol.



(i) Distance = Length of actual circular path from A to B = Half the circumference

i.e. Distance = $\frac{2\pi r}{2} = \pi r$

r = 1m

$\therefore$ Distance = π m

(ii) Displacement = 2r along west = 2m along west

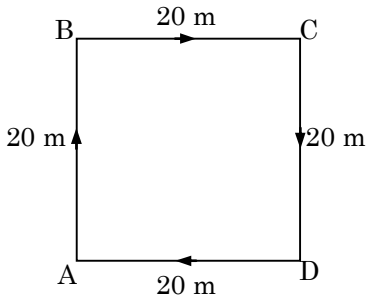
Ex.12 What does the odometer of an automobile measure?

Sol. The odometer of an automobile measures the distance covered by an automobile.

Ex.13 An object has moved through a distance. Can it have zero displacement? If yes, support your answer with an example.

Sol. Yes, an object that has moved through a distance can have zero displacement. Displacement is the shortest measurable distance between the initial and the final position of an object. An object which has covered a distance can have zero displacement, if it comes back to its starting point, i.e., the initial position.

Consider the following situation: A man is walking in a square park of length 20 m (as shown in the following figure). He starts walking from point A and after moving along all the corners of the park (point B, C, D), he again comes back to the same point, i.e., A.



In this case, the total distance covered by the man is 20 m + 20 m + 20 m + 20 m = 80 m. However, his displacement is zero because the shortest distance between his initial and final position is zero.

Ex.14 A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds?

Sol. The farmer takes 40 s to cover $4 \times 10 = 40$ m.

In 2 min and 20 s (140 s), he will cover a distance $\frac{40}{40} \times 140 = 140$ m

Therefore, the farmer completes $\frac{140}{40} = 3.5$ rounds (3 complete rounds and a half round) of the field in 2 min and 20 s.

That means, after 2 min 20 s, the farmer will be at the opposite end of the starting point.

Now, there can be two extreme cases.

Case I: Starting point is a corner point of the field.

In this case, the farmer will be at the diagonally opposite corner of the field after 2 min 20 s.

Therefore, the displacement will be equal to the diagonal of the field.

Hence, the displacement will be $\sqrt{10^2 + 10^2} = 14.1$ m

Case II: Starting point is the middle point of any side of the field.

In this case the farmer will be at the middle point of the opposite side of the field after 2 min 20 s.

Therefore, the displacement will be equal to the side of the field, i.e., 10 m.

For any other starting point, the displacement will be between 14.1 m and 10 m.

Speed & Velocity

◈ Speed

The distance travelled by a body in unit time is called its speed. Therefore, speed = $\frac{\text{Distance}}{\text{Time}}$ or $s = \frac{d}{t}$

S.I. unit of speed or average speed is m/sec. It is a scalar quantity.

◈ Types of Speed

- **Average Speed:** For an object moving with variable speed, it is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(i) Let initial speed of an object is v_1, final speed is v_2 and acceleration is constant, then

average speed = $\frac{v_1 + v_2}{2}$

(ii) A body covers a distance s_1 in time t_1, s_2 in time t_2 and s_3 in time t_3.

Then, average speed, $v_{av} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

(iii) A body travels with speed v_1 for a time t_1 v_2 for time t_2 and v_3 for the time t_3.

Then, average speed, $v_{av} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3}{t_1 + t_2 + t_3}$

$s_1 = v_1 t_1$, $s_2 = v_2 t_2$ and $s_3 = v_3 t_3$

if $t_1 = t_2 = t_3 = t$

$v_{av} = \frac{t(v_1 + v_2 + v_3)}{3t}$

$v_{av} = \frac{(v_1 + v_2 + v_3)}{3}$

(iv) A body covers a distance s_1 with speed v_1 , s_2 with speed v_2 and s_3 with speed v_3.

Then, average speed, $v_{avg} = \frac{(s_1 + s_2 + s_3)}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}}$

$t_1 = \frac{s_1}{v_1}$, $t_2 = \frac{s_2}{v_2}$, $t_3 = \frac{s_3}{v_3}$

(v) A boy goes from home to school with speed v_1 and come back to home with speed v_2.

Here distance covered by the boy is same

Time taken by the boy, while traveling from home to school,

$t_1 = \frac{s}{v_1}$

and time taken by the boy, while traveling from school to home,

$t_2 = \frac{s}{v_2}$

Then, average speed, $v_{av} = \frac{s + s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}}$

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

(vi) If an object covers 1/3rd distance with speed u, next 1/3rd with speed v and last 1/3rd distance with speed w, then $v_{avg} = \frac{3uvw}{uv + vw + wu}$

- **Uniform Speed (or Constant Speed):** When an object covers equal distance in equal intervals of time, it is said to move with uniform speed. e.g.: A car moves 10 m in every one second so its motion is uniform.
- **Variable Speed (Non-Uniform Speed):** If a body covers unequal distance in equal intervals of time, its motion is said to be non-uniform. e.g.: Falling of an apple from a tree, a cyclist moving on a rough road, an athlete running a race, vehicle starting from rest, the motion of a freely falling body etc.
- **Instantaneous speed:** The speed of object at a particular instant is called instantaneous speed.

COMPETITIVE LEVEL

The limiting value of average speed when the time interval approaches zero, Thus,

Instantaneous speed = $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

◈ Velocity

It is defined as the rate of change of displacement.

Therefore, velocity = $\frac{\text{displacement}}{\text{time}}$

or it is the distance travelled in unit time in a given direction.

Velocity = $\frac{\text{Distance travelled in a given direction}}{\text{time taken}}$

S.I. unit of velocity is m/s. It is a vector quantity.

(Magnitude of the velocity is known as speed)

Note:
- *To convert m/s into km/h we multiply by 18/5*
- *To convert km/h into m/s we multiply by 5/18*

◈ Types of Velocity

- **Uniform Velocity (Constant Velocity):** If a body covers equal distance in equal intervals of time in a given direction then it is said to be moving with constant velocity.
- **Non-Uniform Velocity:** When a body does not cover equal distances in equal intervals of time, in a given direction (in this case speed is not constant), then it is known as non uniform velocity.

  In uniform circular motion speed is constant but velocity is not constant.
- **Average Velocity:** It is defined as the ratio of total displacement to the total time taken for this displacement. It is denoted by $\bar{v}_{av}$ **or** $\bar{v}$. It is a vector quantity.

  $\therefore$ Average velocity = $\frac{\text{Total displacement}}{\text{total time}}$

  i.e. $\bar{v} = \frac{s}{t}$

  It is a vector quantity, and its direction is in the direction of displacement.

Note:
- *If for a straight line motion displacement is positive then $\bar{v}$ is positive.*
- *If displacement is negative then $\bar{v}$ is also negative.*

- **Instantaneous Velocity:** The velocity of an object at a particular instant of time is called instantaneous velocity. It is a vector quantity.

COMPETITIVE LEVEL

It is equal to the limiting value of average velocity of the object when the time interval approaches zero, thus Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$

◈ **More about Speed and Velocity:**

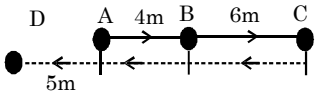
(i) The instantaneous velocity in magnitude is equal to instantaneous speed.

(ii) A particle may have constant speed but variable velocity. In uniform circular motion speed remains constant while velocity changes because of change in direction in motion.

(iii) If particle is moving along a straight line without changing the direction, its average velocity will be equal to its average speed. Otherwise average velocity will be less than average speed.

$$\left|\frac{\text{Average velocity}}{\text{average speed}}\right| \leq 1$$

Ex.15 A body starts from A and moves according to given figure. Time for each interval is:
t_{AB} = 2s, t_{BC}= 3s, t_{CB}=2s, t_{BA} = 3s, t_{AD}= 4s



Find the distance, displacement, speed and velocity for path AB, ABC, ABCB, ABCA, ABCAD.

Sol. The distance, displacement, speed and velocity are as follows for different paths.

Path	Distance	Displacement	Speed	Velocity
AB	4m	4m	4/2 m/s	4/2 m/s
ABC	10m	10m	10/5 m/s	10/5 m/s
ABCB	16m	4m	16/7 m/s	4/7 m/s
ABCA	20m	0m	20/10 m/s	0/10 m/s
ABCAD	25m	–5m	25/14 m/s	–5/14 m/s

Ex.16 Distinguish between speed and velocity.

Sol. Difference between speed and velocity:

Speed	Velocity
1. It is the rate of change of position of an object.	1. It is the rate of change of position of an object in a specific direction.
2. Speed= $\frac{\text{distance travelled}}{\text{time}}$	2. Velocity = $\frac{\text{displacement}}{\text{time}}$
3. It is a scalar quantity.	3. It is a vector quantity
4. Speed will always be positive	4. It will be positive or negative depending on the direction of motion.
5. For moving body, it will never be zero.	5. It may be zero.

Ex.17 What does the path of an object look like when it is in uniform motion?

Sol. An object having uniform motion has a straight line path.

Ex.18 During an experiment, a signal from a spaceship reached the ground station in five minutes. What is the distance of the spaceship from the ground station? The signal travels at the speed of light, that is, 3×10^8 m s^{-1}.

Sol. Time taken by the signal to reach the ground station from the spaceship = 5 min = $5 \times 60 = 300$ s
Speed of the signal = 3×10^8 m/s

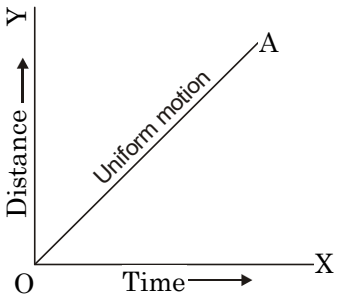
$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$\therefore$ Distance travelled = Speed × Time taken = $3 \times 10^8 \times 300 = 9 \times 10^{10}$
Hence, the distance of the spaceship from the ground station is 9×10^{10} m.

Uniform and Non-Uniform Motion

◈ Uniform Motion

A body has a uniform motion if it travels equal distances in equal intervals of time, no matter how small these time intervals may be. For example, a car running at a constant speed say, 10 metre per second, will cover equal distances of 10 metre every second, so its motion will be uniform. Please note that the distance-time graph for uniform motion is a straight line (As shown in the figure).



◈ Non-Uniform Motion

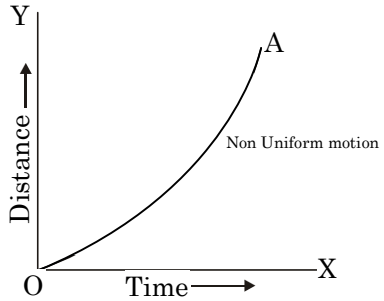
A body has a non-uniform motion if it travels unequal distances in equal intervals of time. For example, if we drop a ball from the roof of a building, we will find that it covers unequal distances in equal intervals of time. It covers:

4.9 metre in the 1st second,

14.7 metre in the 2nd second,

24.5 metre in the 3rd second, and so on.

Thus, a freely falling ball covers smaller distance in the initial '1 second' interval and larger distance in the later '1 second' interval. From this discussion we conclude that the motion of a freely falling body is an example of non-uniform motion. The motion of a train starting from the railway station is also an example of non-uniform motion. This is because when the train starts from a station, it moves a very small distance in the 'first' second. The train moves a little more distance in the '2nd' second and so on. And when the train approaches the next station, the distance travelled by it per second decreases.



Please note that the distance-time graph for a body having non-uniform motion is a curved line (As shown in the figure). Thus, in order to find out whether a body has uniform motion or non-uniform motion, we should draw the distance-time graph for it. If the distance time graph is a straight line, the motion will be uniform and if the distance-time graph is a curved line, the motion will be non-uniform. It should be noted that non-uniform motion is also called accelerated motion.

◈ Features of Uniform Motion

(i) The velocity in uniform motion does not depend on the choice of origin.

(ii) The velocity in uniform motion does not depend on the choice of the time interval.

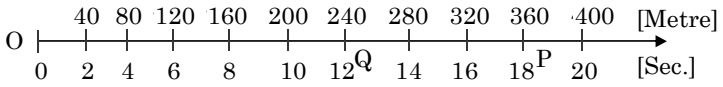
(iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

(iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(v) For an object in uniform motion no force is required to maintain its motion.

(vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant, at each point of the path.

Ex.19 A car is moving along x-axis. As shown in figure it moves from O to P in 18 s and returns from P to Q in 6 second. What is the average velocity and average speed of the car in going from (i) O to P and (ii) from O to P and back to Q.



Sol. (i) Average velocity $= \frac{\text{Displacement}}{\text{time interval}} = \frac{360}{18} = 20 \text{ms}^{-1}$

Average speed $= \frac{\text{path length}}{\text{time interval}} = \frac{360}{18} = 20 \text{ ms}^{-1}$

(ii) From O to P and back to Q

Average velocity $= \frac{OQ}{18+6} = \frac{240\text{m}}{24} = 10 \text{ ms}^{-1}$

Average speed $= \frac{\text{path length}}{\text{time interval}}$

$= \frac{OP+PQ}{18+6} = \frac{360+120}{24} = 20 \text{ ms}^{-1}$

Ex.20 A car covers the 1st half of the distance between two places at a speed of 40 km h^{-1} and the 2nd half with 60 km h^{-1}. What is the average speed of the car?

Sol. Suppose the total distance covered is 2S.

Then time taken to cover the distance 'S' with speed 40 km/h,

$$t_1 = \frac{S}{40}\,h$$

Time taken to cover the next distance 'S' with speed 60 km/h,

$$t_2 = \frac{S}{60}\,h$$

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{S+S}{\left(\frac{S}{40}+\frac{S}{60}\right)}$$

$$V_{av} = \frac{2S}{\left(\frac{3S+2S}{120}\right)} = \frac{2S}{5S}\times 120$$

$$\Rightarrow\ V_{av} = 48\ km/h$$

Ex.21 A non-stop bus goes from one station to another station with a speed of 54 km/h, the same bus returns from the second station to the first station with a speed of 36 km/h. Find the average speed of the bus for the entire journey.

Sol. Suppose the distance between the stations is S. Time taken in reaching from one station to another station.

$$t_1 = \frac{S}{54}\,h$$

Time taken in returning back,

$$t_2 = \frac{S}{36}\,h$$

Total time $t = t_1 + t_2$

$$t = \frac{S}{54}+\frac{S}{36} = \frac{2S+3S}{108} = \frac{5S}{108}\,h$$

Average speed $V_{av} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$V_{av} = \frac{2S}{5S}\times 108$$

$$V_{av} = \frac{216}{5} = 43.2\ km/h$$

Acceleration

Mostly the velocity of a moving object changes either in magnitude or in direction or in both when the object moves. The body is then said to have acceleration. So it is the rate of change of velocity i.e. change in velocity in unit time is said to be acceleration. It is a vector quantity and

Its S.I unit is m/s^2 and c.g.s unit is cm/s^2.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{v-u}{t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

◈ Types of Acceleration

- **Uniform Acceleration (Uniformly Accelerated Motion):** If a body travels in a straight line and its velocity increases in equal amounts in equal intervals of time. Its motion is known as uniformly accelerated motion.

  e.g.: Motion of a freely falling body is an example of uniformly accelerated motion (or motion of a body under the gravitational pull of the earth) & motion of a bicycle going down the slope of a road when the rider is not pedaling and wind resistance is negligible.

- **Non-Uniform Acceleration:** If during motion of a body its velocity increases by unequal amounts in equal intervals of time, then its motion is known as non uniform accelerated motion.

  e.g.: Car moving in a crowded street & motion of a train leaving or entering the platform.

- **Positive acceleration:** If the velocity of an object increases with respect to time in the same direction, the object has a positive acceleration.

- **Negative acceleration (retardation):** If the velocity of a body decreases with respect to time in the same direction, the body has a negative acceleration or it is said to be retarding.

  e.g.: A train slows down, then its acceleration will be negative.

Ex.22 A bus decreases its speed from 80 km h^{-1} to 60 km h^{-1} in 5 s. Find the acceleration of the bus.

Sol. Initial speed of the bus, u = 80 km/h = $80 \times \frac{5}{18}$ = 22.22 m/s

Final speed of the bus, v = 60 km/h = $60 \times \frac{5}{18}$ = 16.66 m/s

Time take to decrease the speed, t = 5 s

Acceleration a = $\frac{v-u}{t} = \frac{16.66-22.22}{5}$ = –1.112 m/s^2

Here, the negative sign of acceleration indicates that the velocity of the car is decreasing.

Ex.23 A train starting from a railway station and moving with uniform acceleration attains a speed 40 km h^{-1} in 10 minutes. Find its acceleration.

Sol. Initial velocity of the train, u = 0 (since the train is initially at rest)

Final velocity of the train, v = 40 km/h = $40 \times \frac{5}{18}$ = 11.11 m/s

Time taken, t = 10 min = 10 × 60 = 600 s

Acceleration a = $\frac{v-u}{t} = \frac{11.11-0}{600}$ = 0.0185 m/s^2

Hence, the acceleration of the train is 0.0185 m/s^2.

Graphical Representation of Motion

◈ Distance–Time Graph

A moving body changes its position continuously with time. The simplest way to describe the motion of a moving body is to draw its distance–time graph.

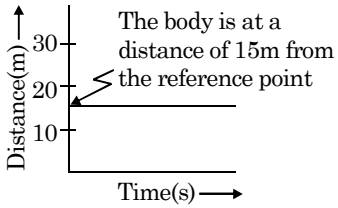
The distance–time graphs of a body under the following three conditions are described below:

(i) When the body is at rest.

(ii) When the body is moving with a uniform speed

(iii) When the body is moving with a non–uniform speed

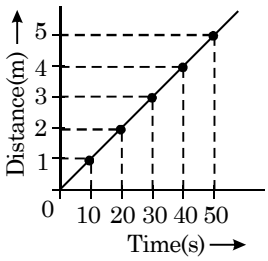
- Distance–time graph for a body at rest



The distance–time graph for a body at rest is a straight line parallel to the time axis

- Distance–time graph for a body moving with a uniform speed:

When a body covers equal distances in equal intervals of time, it is said to have uniform speed.



Distance–time graph of a body moving with uniform (Constant) speed

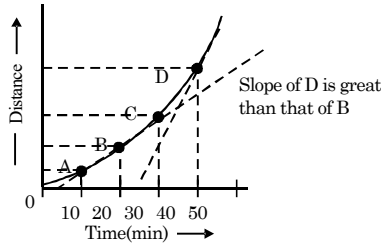
The about graph shows that the distance travelled by a body moving with uniform speed is directly proportional to time.

- Distance–time graph for a body moving with a non–uniform speed:

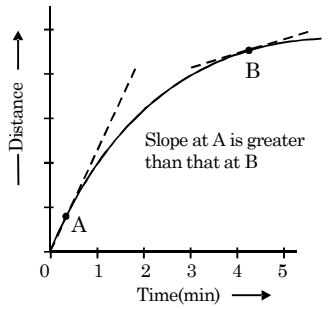
A body moving with a non–uniform speed covers unequal distances in equal intervals of time. Therefore, the distance–time graph of a body moving with a non–uniform speed is a curve.

The shape of the distance–time graph for a body moving with non–uniform speed depends upon the way speed of the body changes with time. Two typical cases are described below:

(i) When the speed increases with time: When the speed of a body increases with time, the distance covered by it in one unit of time also increases with time. Therefore, the distance–time graph for a body moving with an increasing non–uniform speed is a curve whose slope increases with time (Figure).

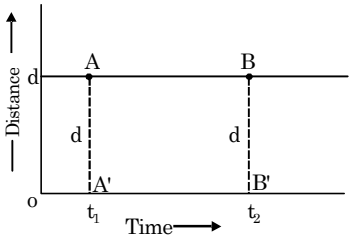


(ii) When the speed decreases with time: When the speed of a body decreases with time, the distance covered by it in one unit of time also decreases with time. Therefore, the distance–time graph for a body moving with a decreasing non–uniform speed is a curve whose slope decreases with time (Figure).



◈ Displacement-Time Graph

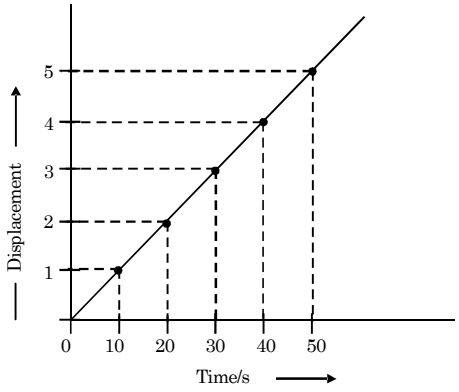
- **Displacement – time graph of a body at rest:** The position of a body at rest remains unchanged with time. Let us consider a body at a distance d from a reference point in a particular direction. Then from figure 4.1.



The above graph shows that position of the body does not change w.r.t. time, so that body is said to be at rest.

Thus, the velocity of a body at rest is zero.

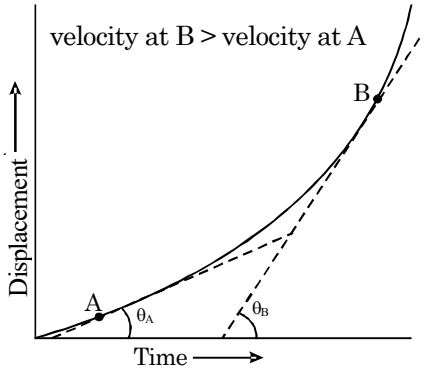
- **Displacement – time graph of a body moving with uniform velocity:** The displacement–time graph of a body moving with uniform (constant) velocity is a straight line inclined to the time–axis at certain angle.



The slope of the displacement–time graph for a body moving with uniform velocity is equal to the velocity of the body.

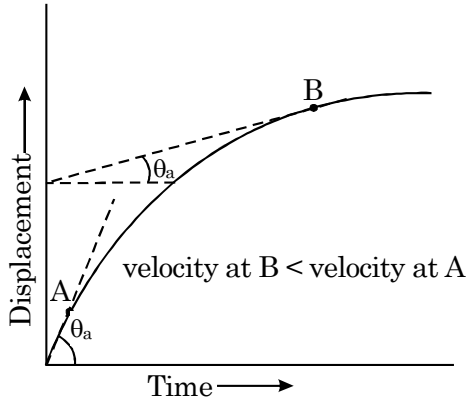
- **Displacement–time graph of a body moving with an increasing non–uniform velocity :** The displacement–time graph of a body moving with an increasing non–uniform velocity is a curve (Figure). Here, the slope of the curve increases with time. So, the velocity of the body increases with time.

  i.e., velocity at B > velocity at A



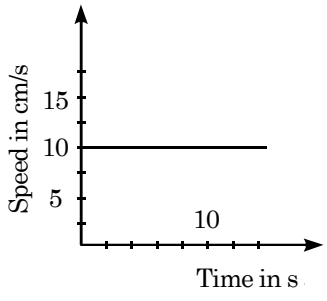
- **Displacement–time graph of a body moving with decreasing non–uniform velocity :** The displacement–time graph of a body moving with a decreasing non–uniform velocity is a curve (Figure). Here, the slope of the curve decreases with time. So, the velocity of the body decreases with time.

  i.e., velocity at B < velocity at A



◈ Speed-Time Graph

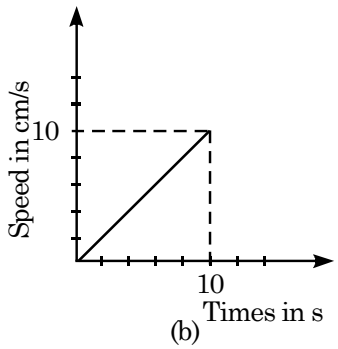
Some information about the motion of an object can also be obtained from its speed–time graph. Following figures give the speed–time graphs of four different objects in motion. Let us see what information can be obtained from these graphs.



(a)

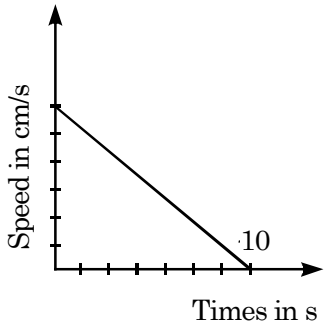
Figure (a) shows that for any time, the speed has the same value (10 cm/s). Thus it represents an object moving with a constant speed.

Whenever an object moves with a constant speed, its speed–time graph is a straight line, parallel to the time–axis.



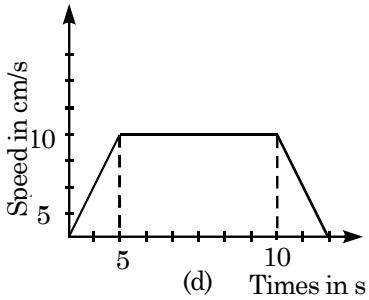
(b)

Figure (b) shows that the speed continuously increases with time. At time t = 0, the speed is 0. At t = 10 s, it becomes 10 cm/s. The straight–line nature of the graph indicates that the speed increases at a constant rate.



(c)

Figure (c) shows that the speed is 10 cm/s at t = 0 and gradually decreases as time passes. Thus it represents a decelerating object. Here also the speed changes at constant rate. At t = 10s, the speed becomes zero.



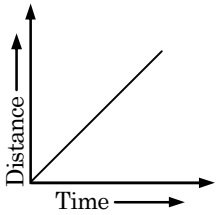
(d)

Figure (d) represents the motion of an object which speeds up from t = 0 to t = 5s, then moves at a constant speed from t = 5s to 10s and then decelerates to stop at t = 15 s.

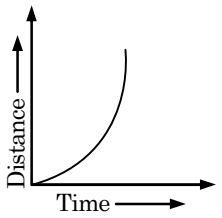
Note: *Distance covered by the body is shown by the area under speed–time graph.*

Ex.24 What is the nature of the distance - time graphs for uniform and non-uniform motion of an object?

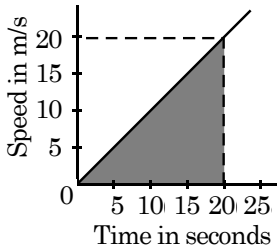
Sol. The distance–time graph for uniform motion of an object is a straight line (As shown in the following figure).



The distance - time graph for non-uniform motion of an object is a curved line (As shown in the given figure).



Ex.25 Find the distance covered by a particle during the time interval t = 0 to t = 20s for which the speed–time graph is shown in figure.

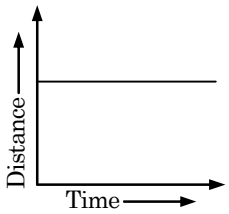


Sol. The distance covered in the time interval 0 to 20s is equal to the area of the shaded triangle. It is

$\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times (20\text{s}) \times (20\text{ m/s}) = 200$ m.

Ex.26 What can you say about the motion of an object whose distance-time graph is a straight line parallel to the time axis?

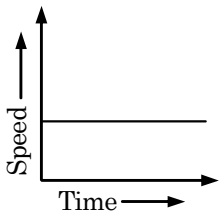
Sol. When an object is at rest, its distance-time graph is a straight line parallel to the time axis.



A straight line parallel to the x-axis in a distance - time graph indicates that with a change in time, there is no change in the position of the object. Thus, the object is at rest.

Ex.27 What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis?

Sol. Object is moving uniformly.



A straight line parallel to the time axis in a speed-time graph indicates that with a change in time, there is no change in the speed of the object. This indicates the uniform motion of the object.

◈ Velocity-Time Graph

If a graph is plotted taking the velocity of an object moving along a straight line on the vertical axis and time on the horizontal axis, we get a velocity–time graph.

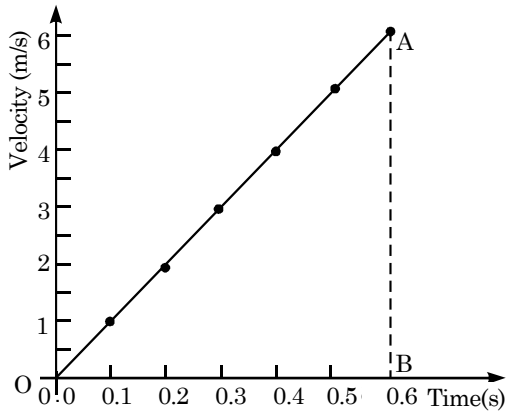
Suppose an object moves along a straight line in a fixed direction. That means the object does not turn around during its motion. Taking the direction of motion as the positive direction, the velocity of the object is given by the same value as its speed. Thus the speed–time graph for such an object is also its velocity–time graph.

The area under the speed–time graph gives the distance covered. But for a particle moving in a fixed direction, the distance covered in a time interval has the same value as its displacement in that time interval. So, the area under the velocity–time graph of an object gives its displacement.

Let us take an example. A ball is dropped from a height. We take the downward direction as positive. As the ball falls, its velocity increases. The velocity of the ball at different instants are given in Table-1. The velocity versus time graph is shown in figure.

Table-1: Velocity of the falling ball at different instants:

Time in s	Velocity in m/s
0	0
0.1	1
0.2	2
0.3	3
0.4	4
0.5	5
0.6	6



What is the displacement of the ball in the time interval 0 to 0.6s? It is equal to the area under the velocity–times graph from t = 0 to t = 0.6 s. This area is in the shape of a triangle. The area is

$\frac{1}{2}$ × base × height

= $\frac{1}{2}$ (OB) × (AB)

= $\frac{1}{2}$ × (0.6s) × (6 m/s) = 1.8 m.

The ball has fallen through 1.8 m in 0.6 s.

- **Velocity–time graph of an object thrown upwards:** Suppose a ball is thrown upwards. We take the upward direction as the positive direction. The velocity decreases as the ball goes up. Table-2 gives the velocity of the ball at different instants.

Table-2: Velocity of the rising ball at different instants:

Time(s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20

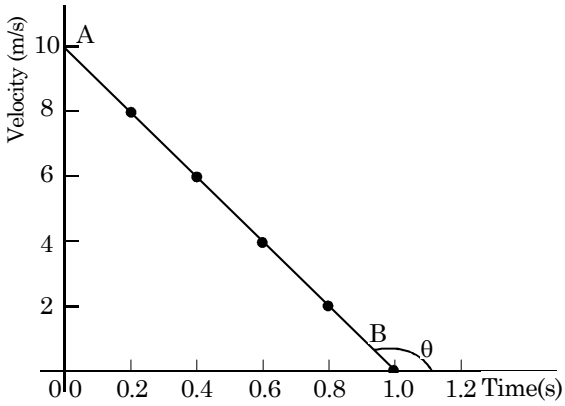
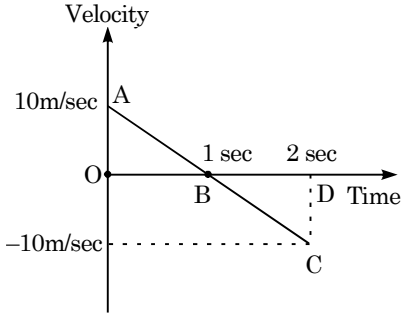


Figure shows the velocity–time graph. The plotted points fall on a straight line, AB. At t = 1.0 s, the velocity becomes zero. This means that the ball reaches the highest point at t = 1.0 s.

Ex.28 A ball is thrown vertically upwards with a velocity of 10m/sec. It strikes the ground after 2 sec. Its velocity–time graph is as shown in figure below. Find the displacement travelled by the ball in 2 second.



Sol. Displacement = area under velocity-time curve along time axis

= area of triangle AOB + area of triangle BDC

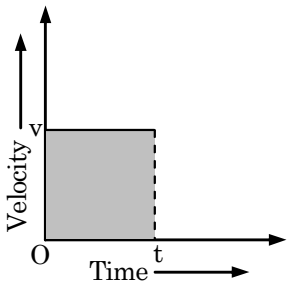
$$= \left(\frac{1}{2} \times OB \times AO\right) + \left(\frac{1}{2} \times BD \times CD\right)$$

$$= \left(\frac{1}{2} \times 1\,\text{sec} \times 10\,\text{m/sec}\right) + \left(\frac{1}{2} \times 1\text{sec} \times (-10\text{m/sec})\right)$$

= 5m –5m = 0m

Ex.29 What is the quantity which is measured by the area occupied below the velocity - time graph?

Sol. Displacement



The graph shows the velocity-time graph of a uniformly moving body.
Let the velocity of the body at time (t) be v.
Area of the shaded region = length × breath
Where, Length = t; Breath = v
Area = vt = velocity × time …(i)
We know, velocity = displacement / time
∴ Displacement = Velocity × Time …(ii)
From equations (i) and (ii), Area = Displacement

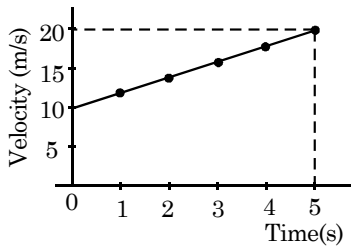
Hence, the area occupied below the velocity-time graph measures the displacement covered by the body.

- **Acceleration from velocity–time graph:** Suppose a particle moves with a uniform acceleration of 2 m/s^2 along a straight line. This means that the velocity increases by 2 m/s in one second. Also suppose its speed at t = 0 is 10 m/s.

  Let us plot the velocity–time graph for this situation. We first find the values of the velocity at certain instants. At t = 0, the velocity is 10 m/s, at t = 1s it will become 10 m/s + 2m/s = 12 m/s, at t = 2s, it will become 14 m/s and so on. These values are given in Table-3 and the velocity–time graph is shown in Figure.

Table-3: Velocity at different instants

Time(s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20



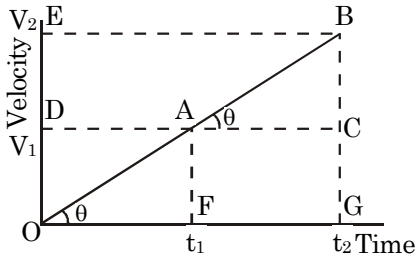
We see that the graph is a straight line. Whenever the acceleration is uniform the velocity–time graph is a straight line. We will now show that the slope of the velocity–time graph gives the acceleration.

Suppose the velocity–time graph of a particle moving along a straight line is as shown in figure. The graph is a straight line. At time t_1, the velocity is v_1, and at time t_2, it is v_2. These values are represented by the points A and B on the graph.

The acceleration of the object is

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{OE - OD}{OG - OF} = \frac{DE}{FG} = \frac{BC}{AC} = \tan\theta$$

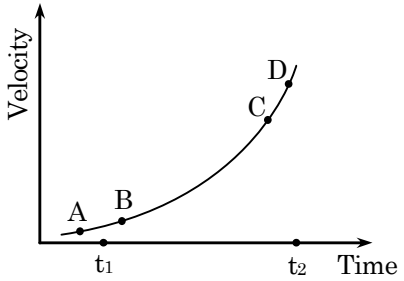
where, θ is the angle made by the graph with the time-axis.



As defined earlier, the ratio $\frac{BC}{AC} = \tan\theta$ is called the slope of the line. Thus, we have the following:

The slope of the velocity–time graphs gives the acceleration for an object moving along a straight line.

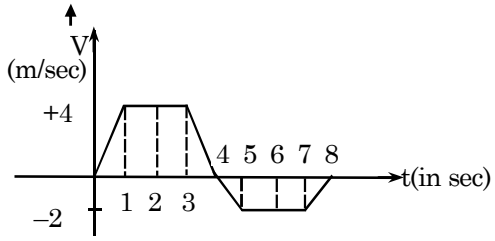
- **Non–uniform acceleration:** If the acceleration of an object moving along a straight line is not constant, the velocity-time graph is not a straight line. Consider the velocity–time graph shown in figure. To find the acceleration at time t_1 we should treat the small part AB as a straight line and find its slope. Similarly, the slope of the small part CD gives the acceleration at time t_2. It is clear from the figure that the slope of CD is greater than that of AB. Thus, the acceleration at t_2 is greater than that at t_1. The graph in figure represents a motion in which acceleration increases with time.



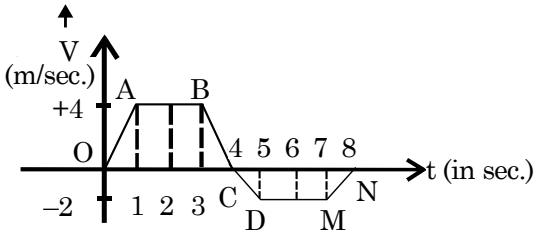
Displacement of particle from velocity time graph.

The area under the velocity–time curve along time axis gives the displacement of the particle.

Ex.30 The velocity versus time graph of a linear motion is shown in figure. Find the distance from the origin in 8 second.



Sol.



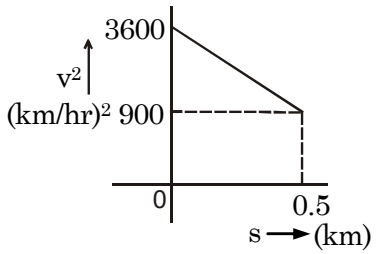
Distance in 8 second

S = Area of OABC + Area of CDMN

$$S = \frac{(2+4)\times 4}{2} + \frac{(2+4)\times 2}{2}$$

S = 18 m

Ex.31 A graph between the square of the velocity of a particle and the distance moved by the particle is as shown in the figure. Find the acceleration of the particle in km/h^2.



Sol. Given : $u^2 = 3600$ (km/h)^2, $v^2 = 900$ (km/h)^2,

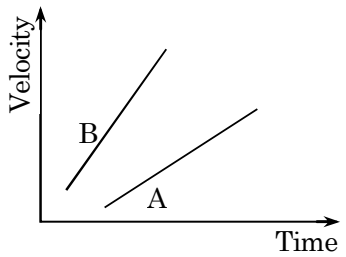
s = 0.5 km

From III equation of motion,

$v^2 = u^2 + 2as \quad \Rightarrow a = \frac{v^2 - u^2}{2s}$

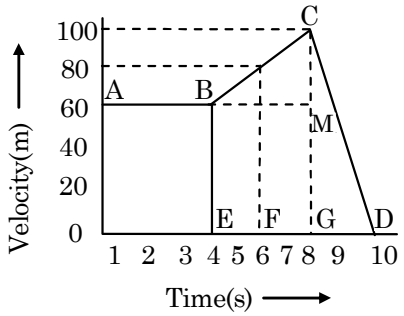
$$\Rightarrow a = \frac{(900)-(3600)}{2\times 0.5} = \frac{-2700}{2\times 0.5} = -2700 \text{ km/h}^2$$

Ex.32 Figure shows the velocity–time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration?



Sol. The slope of the velocity–time graph for B is greater than that for A. Thus, the acceleration of B is greater than that of A.

Ex.33 Time-velocity graph of a particle is shown in figure. Find its instantaneous acceleration at following intervals:



(i) at t = 3s (ii) at t = 6s (iii) at t = 9s

Sol. (i) Instantaneous acceleration at t = 3s, is given by

a = slope of line AB = zero

(ii) Instantaneous acceleration at t = 6 s, is given by a = slope of line

$$BC = \frac{CM}{BM} = \frac{100-60}{8-4} = 10 \text{ m/s}^2$$

(iii) Instantaneous acceleration at t = 9 s is given by, a = slope of line $CD = \frac{0-100}{10-8} = -50 \text{ m/s}^2$

Ex.34 When will you say a body is in

(i) uniform acceleration? (ii) non-uniform acceleration?

Sol. (i) A body is said to have uniform acceleration if it travels in a straight path in such a way that its velocity changes at a uniform rate, i.e., the velocity of a body increases or decreases by equal amounts in an equal interval of time.

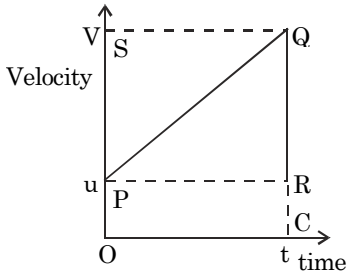
(ii) A body is said to have non-uniform acceleration if it travels in a straight path in such a way that its velocity changes at a non-uniform rate, i.e., the velocity of a body increases or decreases in unequal amounts in an equal interval of time.

Equations of Uniformly Accelerated Motion

◈ Graphical Derivation of Equations of Motion

- **First Equation:** v = u + at

It can be derived from v– t graph, as shown in figure: From line PQ, the slope of the line = acceleration



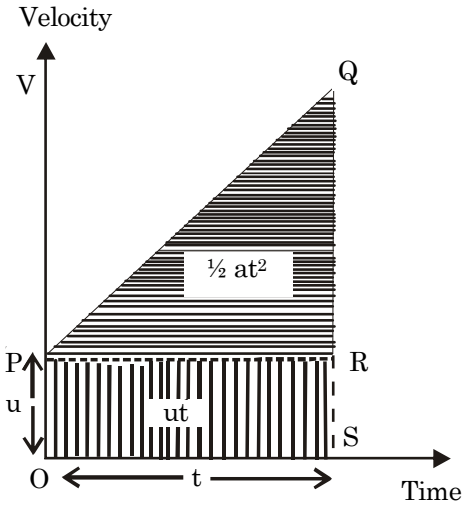
$$a = \frac{QR}{RP} = \frac{SP}{RP}$$

$\because$ SP = v – u and RP = t

So $a = \frac{v-u}{t}$ **or** v = u + at

- **Second Equation:** $s = ut + \frac{1}{2} at^2$

It can also be derived from v - t graph as shown in figure. From relation, Distance covered = Area under v - t graphs = Area of trapezium OPQS = Area of rectangle OPRS + Area of triangle PQR



$= OP \times PR + \frac{RQ \times PR}{2}$

Putting values,

$s = u \times t + \frac{1}{2} (v - u) \times t$ $\quad (\because RQ = v - u \ \& \ PR = OS = t)$

$= u \times t + \frac{1}{2} at \times t$ $\quad (\because v - u = at)$

or $s = ut + \frac{1}{2} at^2$

- **Third Equation :**

$v^2 = u^2 + 2as$

From above graph OP = u, SQ = v, OP + SQ = u + v

$a = \frac{QR}{PR}$ Or $PR = \frac{QR}{a} = \frac{v - u}{a}$

s = Area of trapezium OPQS = $\frac{OP + SQ}{2} \times PR$

On putting the values,

$s = \frac{u + v}{2} \times \frac{v - u}{a} = \frac{v^2 - u^2}{2a}$

or $v^2 = u^2 + 2as$

◈ Analytical Derivation for Equations of Uniformly Acceleration Motion

There are three equation of uniformly accelerated motion. They show the relation between initial velocity u, final velocity v, acceleration a, time t and displacement s

- **1st Equation of Motion:** Consider a body moving with initial velocity u and its velocity changes from u to v in time t. Then

  $$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

  $$\Rightarrow \quad a = \frac{v - u}{t}$$

  So $at = v - u$ and $v = u + at$

  1st equation of motion: $v = u + at$

- **2nd Equation of Motion:** We know

  Distance covered = (Average velocity) × (Time)

  or $$s = \frac{u + v}{2} t$$

  But $v = u + at$

  Substituting the value of v in the equation above, we have

  $$s = \frac{u + (u + at)}{2} t$$

  or $$s = \left(\frac{2u + at}{2}\right) t = \left(u + \frac{at}{2}\right) t$$

  or $$s = ut + \frac{1}{2} at^2$$

  2nd equation of motion: $s = ut + \frac{1}{2} at^2$

- **3rd Equation of Motion:** We know that

  $$v = u + at$$

  or $$t = \frac{v - u}{a}$$

  Distance travelled = (Average velocity) × (time)

  $$s = \left(\frac{v + u}{2}\right) t = \left(\frac{v + u}{2}\right)\left(\frac{v - u}{a}\right)$$

  or $s = \frac{v^2 - u^2}{2a}$ or $v^2 - u^2 = 2as$

  3rd equation of motion: $v^2 - u^2 = 2as$

- **Distance covered in n^{th} second:** Distance travelled in n^{th} second = Distance travelled in n sec – Distance travelled in (n –1) sec.

  So, $S_{nth} = S_n - S_{(n-1)}$

  $$\left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

  [Putting t = n and t = (n – 1) respectively in equation (ii)]

  $= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 - 2n + 1)$

  We have, $S_{nth} = u + \frac{a}{2}(2n - 1)$

Ex.35 A bus starting from rest moves with a uniform acceleration of 0.1 m s^{-2} for 2 minutes. Find

(i) the speed acquired, (ii) the distance travelled.

Sol. (i) Initial speed of the bus, u = 0 (since the bus is initially at rest)

Acceleration, a = 0.1 m/s^2

Time taken, t = 2 minutes = 120 s

Let v be the final speed acquired by the bus.

$\therefore \quad a = \frac{v - u}{t}$

$0.1 = \frac{v - 0}{120}$

$\therefore \quad v = 12$ m/s

(ii) According to the third equation of motion:

$v^2 - u^2 = 2as$

where, s is the distance covered by the bus $(12)^2 - (0)^2 = 2(0.1)$ s

s = 720 m

Speed acquired by the bus is 12 m/s.

Distance travelled by the bus is 720 m.

Ex.36 A train is travelling at a speed of 90 km h^{-1}. Brakes are applied so as to produce a uniform acceleration of −0.5 m s^{-2}. Find how far the train will go before it is brought to rest.

Sol. Initial speed of the train, u = 90 km/h = 25 m/s

Final speed of the train, v = 0 (finally the train comes to rest)

Acceleration = −0.5 m s^{-2}

According to third equation of motion: $v^2 = u^2 + 2as$

$(0)^2 = (25)^2 + 2(-0.5)s$

where, s is the distance covered by the train

$s = \frac{(25)^2}{2(0.5)} = 625$ m

The train will cover a distance of 625 m before it comes to rest.

Ex.37 A trolley, while going down an inclined plane, has an acceleration of 2 cm s^{-2}. What will be its velocity 3 s after the start?

Sol. Initial velocity of the trolley, u = 0 (since the trolley was initially at rest)

Acceleration, a = 2 cm s^{-2} = 0.02 m/s^2

Time, t = 3 s

According to the first equation of motion:

v = u + at

where, v is the velocity of the trolley after 3 s from start

v = 0 + 0.02 × 3 = 0.06 m/s

Hence, the velocity of the trolley after 3 s from start is 0.06 m/s.

Ex.38 A racing car has a uniform acceleration of 4 m s^{-2}. What distance will it cover in 10 s after start?

Sol. Initial velocity of the racing car, u = 0 (since the racing car is initially at rest)

Acceleration, a = 4 m/s^2

Time taken, t = 10 s

According to the second equation of motion:

$s = ut + \frac{1}{2} at^2$

where, s is the distance covered by the racing car

$s = 0 + \frac{1}{2} \times 4 \times (10)^2 = \frac{400}{2} = 200$ m

Hence, the distance covered by the racing car after 10 s from start is 200 m.

◈ Equations of Motion for Freely Falling Object

Since the freely falling bodies fall with uniformly accelerated motion, the three equations of motion derived earlier for bodies under uniform acceleration can be applied to the motion of freely falling bodies. For freely falling bodies, the acceleration due to gravity is 'g', so we replace the acceleration 'a' of the equations by 'g' and since the vertical distance of the freely falling bodies is known as height 'h', we replace the distance 's' in our equations by the height 'h'. This gives us the following modified equations for the motion of freely falling bodies.

General equations of motion		**Equations of motion for freely falling bodies**
(i) $v = u + at$	changes to	$v = u + gt$
(ii) $s = ut + \frac{1}{2} at^2$	changes to	$h = ut + \frac{1}{2} gt^2$
(iii) $v^2 = u^2 + 2as$	changes to	$v^2 = u^2 + 2gh$

We shall use these modified equations to solve numerical problems. Before we do that, we should remember the following important points for the motion of freely falling bodies.

(i) When a body is dropped freely from a height, its initial velocity 'u' becomes zero.

(ii) When a body is thrown vertically upwards, its final velocity 'v' becomes zero.

(iii) The time taken by a body to rise to the highest point is equal to the time it takes to fall from the same height.

(iv) The distance travelled by a freely falling body is directly proportional to the square of time of fall.

- **Sign conventions:**

  (i) g is taken as positive when it is acting in the same direction as that of motion and g is taken as negative when it is opposing the motion.

  (ii) Distance measured upward from the point of projection is taken as positive, while distance measured downward from the point of projection is taken as negative.

  (iii) Velocity measured away from the surface of earth (i.e. in upward direction) is taken as positive, while velocity measured towards the surface of the earth is taken as negative.

- **To solve numerical problems:**

  (i) If a body is dropped from a height then its initial velocity u = 0 but has acceleration (Acting). If a body starts from rest its initial velocity u = 0 .

  (ii) If a body comes to rest its final velocity v = 0 or, if a body reaches the highest point after being thrown upwards its final velocity v = 0 but has acceleration (acting).

  (iii) If a body moves with uniform velocity, its acceleration is zero i.e. a = 0.

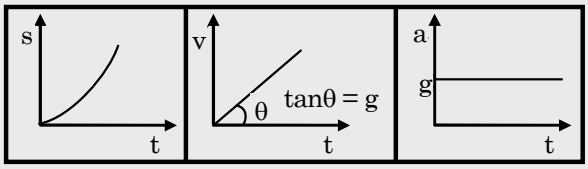
  (iv) Motion of a body is called free fall if only force acting on it is gravity (i.e. earth's attraction).

COMPETITIVE LEVEL

◈ Body Falling Freely Under Gravity

Assuming u = 0 for a freely falling body:

t is given	h is given	v is given
$v = gt$	$t = \sqrt{\frac{2h}{g}}$	$t = \frac{v}{g}$
$h = \frac{1}{2} gt^2$	$v = \sqrt{2gh}$	$h = \frac{v^2}{2g}$

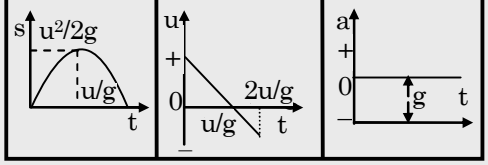


- **Body is projected vertically up:** Taking initial position as origin and direction of motion (i.e. vertically up) as positive.

  (i) At the highest point v = 0

  (ii) a = – g

t is given	h is given	u is given
$u = gt$	$t = \sqrt{2h/g}$	$t = \frac{u}{g}$
$h = \frac{1}{2} gt^2$	$u = \sqrt{2hg}$	$h = \frac{u^2}{2g}$



Ex.39 A stone is thrown in a vertically upward direction with a velocity of 5 m s^{-1}. If the acceleration of the stone during its motion is 10 m s^{-2} in the downward direction, what will be the height attained by the stone and how much time will it take to reach there?

Sol. Initially, velocity of the stone, u = 5 m/s

Final velocity, v = 0 (since the stone comes to rest when it reaches its maximum height) Acceleration of the stone, a = acceleration due to gravity, g = 10 m/s^2 (in downward direction) There will be a change in the sign of acceleration because the stone is being thrown upwards.

Acceleration, a = −10 m/s^2

Let s be the maximum height attained by the stone in time t. According to the first equation of motion:

v = u + at

0 = 5 + (−10) t

$\therefore t = \frac{-5}{-10} = 0.5$ s

According to the third equation of motion: $v^2 = u^2 + 2as$

$(0)^2 = (5)^2 + 2(-10)\ s$

$s = \frac{5^2}{20} = 1.25$ m

Hence, the stone attains a height of 1.25 m in 0.5 s.

Ex.40 A stone drops from the edge of a roof. It passes a window 2 metre high in 0.1 second. How far is the roof above the top of the window?

Sol. Let the distance between the top of the window and the roof be h. This problem can be solved in two stages.

(A) For the journey across the window i.e., from B to C

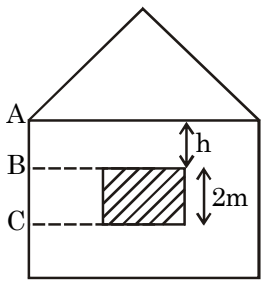
Let, Velocity at B = u m/s

Distance travelled, s = 2 m

Time taken, t = 0.1s

Acceleration, a = g = 9.8 m/s^2

Using the relationship,



$s = ut + \frac{1}{2}gt^2$

$2 = u \times 0.1 + \frac{1}{2} \times 9.8 \times (0.1)^2$

$2 = 0.1\ u + 4.9 \times 10^{-2}$

or, $u = \frac{(2 - 0.049)}{0.1} = 19.51$ m/s

The velocity of the stone at the top of the window is 19.51 m/s.

(B) For journey from roof to the top of the window i.e., from A to B

The velocity at the top of the window is the velocity of the stone at the end of falling through 'h'. So, for this part of the journey,

Initial velocity, u = 0 m/s

Final velocity, v = 19.51 m/s

Acceleration due to gravity, g = 9.8 m/s^2

Then by using the equation, $v^2 - u^2 = 2gh$, one gets

$(19.51)^2 - 0 = 2 \times 9.8 \times h$

$h = \frac{(19.51)^2}{19.6}$ m = 19.42 m

Thus, the roof is 19.42 m from the upper end (top) of the window.

Ex.41 A car is moving at a speed of 50 km/h after two seconds it is moving at 60 km/h. Calculate the acceleration of the car.

Sol. Here u = 50 km/h $= 50 \times \frac{5}{18}$ m/s $= \frac{250}{18}$ m/s

and v = 60 km/h = $60 \times \frac{5}{18} = \frac{300}{18}$ m/s

Since $a = \frac{v - u}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{\frac{50}{18}}{2} = \frac{50}{36} = 1.39$ m/s^2

Ex.42 A body starts moving with an initial velocity 50 m/s and acceleration 20 m/s^2. How much distance it will cover in 4s? Also, calculate its average speed during this time interval.

Sol. Given: u = 50 m/s, a = 20 m/s^2,

t = 4s, s =?

$s = ut + \frac{1}{2}at^2 = 50 \times 4 + \frac{1}{2} \times 20 \times (4)^2$

$= 200 + 160 = 360$ m

Average speed during this interval,

$\overline{V} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{360}{4} = 90$ m/s

Ex.43 A body starts from rest and moves with a constant acceleration. It travels a distance s_1 in first 10 s, and a distance s_2 in next 10 s. Find the relation between s_2 and s_1.

Sol. Given: $u = 0$, $t_1 = 10$ s

$\therefore$ Distance travelled in first 10 seconds, is given by

$s_1 = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times a \times (10)^2$

$= 50a$ .....(1)

To calculate the distance travelled in next 10s, we first calculate distance travelled in 20 s and then subtract distance travelled in first 10 s.

$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times a \times (20)^2$

$= 200a$ ..... (2)

$\therefore$ Distance travelled in next 10 seconds interval,

$s_2 = s - s_1 = 200a - 50a$ ..... (3)

or $s_2 = 150a$

Now, $\frac{s_2}{s_1} = \frac{150a}{50a} = \frac{3}{1}$

or $s_2 = 3s_1$

Ex.44 From the top of a tower of height 490 m, a shell is fired horizontally with a velocity 100 m/s. At what distance from the bottom of the tower, the shell will hit the ground?

Sol. We know that the horizontal motion and the vertical motion are independent of each other. Now for vertical motion, we have $u = 0$, $h = 490$ m, $g = 9.8$ m/s^2, $t = ?$

Using equation, $h = ut + \frac{1}{2} gt^2$, we get

$490 = 0 + \frac{1}{2} \times 9.8 \times t^2$ **or** $t^2 = \frac{490}{4.9} = 100$

or $t = 10$ s

$\therefore$ It takes 10 seconds to reach the ground.

Now, horizontal distance

= horizontal velocity × time

= 100 m/s × 10 s = 1000 m

$\therefore$ The shell will strike the ground at a distance of 100 m from the bottom of the tower.

Circular Motion

◈ Definition

The motion of a body moving around a fixed point in a circular path is known as circular motion.

◈ Uniform Circular motion

If the body covers equal distances along the circumference of the circle in equal intervals of time, the motion is said to be a uniform circular motion. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.

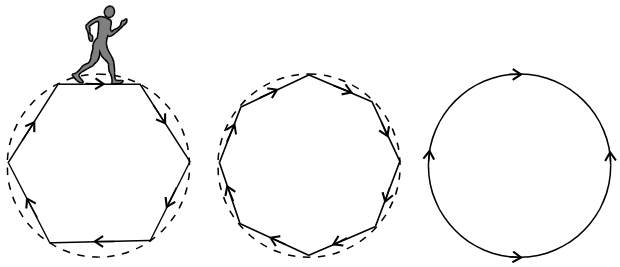
e.g.: Examples of uniform circular motion are:

(i) Motion of moon around the earth.

(ii) Motion of satellite around its planet.

◈ Circular motion is known as accelerated motion

Explanation: Consider a boy running along a regular hexagonal track (path) as shown in figure. As the boy runs along the side of the hexagon at a uniform speed, he has to take a turn at each corner changing direction but keeping the speed same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval will become smaller.



By increasing the number of sides of the regular polygon, we find that number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of a polygon with an infinite number of sides. On the circular track, the turning becomes a continuous process without any gap in between. The boy running along the sides of such a track will be performing a circular motion. Hence, circular motion is the motion of a body along the sides of a polygon of infinite number of sides with uniform speed, the direction changing continuously, it means the body moves with changing velocity in a circular path thus the uniform circular motion is known as accelerated motion.

◈ Difference between a Uniform Linear and Circular Motion

Uniform linear motion	Uniform circular motion
1. The direction of motion does not changes.	**1.** The direction of motion changes continuously.
2. The motion is non accelerated.	**2.** The motion is accelerated.

Note: *Example of a body performing accelerated motion with uniform speed is circular motion.*

COMPETITIVE LEVEL

◈ Radian

It is the unit of plane angle.

- **Definition:** One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to its radius.

  e.g.: In figure, the arc AB of the circle has length ℓ and subtends an angle θ at the centre C.

  If $\angle ACB = \theta$ radians.

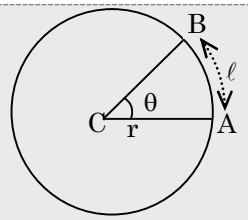
  Then, $\theta = \frac{\ell}{r}$ radians.

  [For $\ell = r$, $\theta = 1$ radian]

  Angle subtended by the circumference at the centre,

  $\theta = \frac{2\pi r}{r} = 2\pi$ radians {or $2\pi^c$ }

  [^{c}] is symbol for radian, just as (^{0}) is symbol for degree.



Relation between radian and degree: For complete circle at centre

$2\pi^c = 360^o$ **or** $1^c = \left|\frac{360}{2\pi}\right| = 57.3^o$

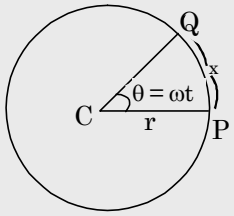
◈ Angular Displacement and Angular Velocity

- **Angular displacement:** In a circular motion, the angular displacement of a body is the angle subtended by the body at the centre in a given interval of time. It is represented by the symbol θ (theta). The unit of angular displacement is radian.
- **Angular velocity:** The angular displacement per unit time is called the angular velocity. It is represented by the symbol ω (omega).
- **Expression for angular displacement and angular velocity:**

(i) Let a body move along a circle of radius r and perform a uniform circular motion. Let the body be at point P to start with and reach point Q after time t. Then, angular displacement = ∠PCQ = θ

angular velocity = $\omega = \frac{\theta}{t}$ (i.e. $\theta = \omega t$)

(ii) In terms of time period and frequency: If the time period of the body is T(time taken in one complete round), the angular displacement = $2\pi^c$



Hence $\omega = \frac{2\pi}{T}$

But $\frac{1}{T} = n$ (frequency)

There $\omega = 2\pi n$

The unit of angular velocity is rad/s

◈ Angular acceleration

The rate of the change of angular velocity is called angular acceleration. Let angular velocity at time t_1 is ω_1 and at time t_2 is ω_2 change in angular velocity in the time interval $t_2 - t_1$ is $\omega_2 - \omega_1$

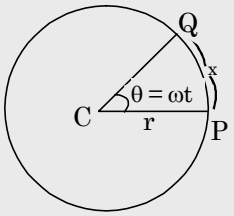
thus, rate of change of angular velocity = $\frac{\omega_2 - \omega_1}{t_2 - t_1}$

$\alpha = \frac{\Delta\omega}{\Delta t}$ (here α is average angular acceleration)

- The unit of angular acceleration is rad/s^2.

◈ Relation between Linear and Angular Quantities

(i) Relation between linear displacement and angular displacement.



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \frac{x}{r} \Rightarrow x = r\theta \qquad \text{..... (i)}$$

(ii) Relation between linear velocity and angular velocity.

From (i) $x = r\theta$

$$\frac{x}{t} = r\frac{\theta}{t} \Rightarrow v = r\omega \qquad \text{..... (ii)}$$

(iii) Relation between linear acceleration and angular acceleration.

From (ii) $v = r\omega$

$$\frac{v}{t} = r\frac{\omega}{t} \Rightarrow a = r\alpha \qquad \text{..... (iii)}$$

◈ Centripetal Force

- Always acts towards centre.
- Centripetal force is required to move a particle in a circle.
- Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

Note: • *Circular motion in horizontal plane is usually uniform circular motion.*

• *Remember that equations of motion are not applicable for circular motion.*

◈ Centripetal Acceleration

- In uniform circular motion the particle experiences an acceleration called the centripetal acceleration.
- $a_c = \frac{v^2}{r}$
- The direction of centripetal acceleration is along the radius towards the centre.

Ex.45 A fly wheel making 120 revolutions/minute. Find the angular speed of the wheel:

Sol. ∵ 120 revolution/ minute = 2 rev/s

Angular speed = angle in one revolution × number of revolution/s

$= 2\pi \times 2 = 4\pi$ rad/s

Ex.46 A stone tied to the end of a string 80 cm is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec., what is the magnitude of the angular speed. $\left(\text{use } \pi = \frac{22}{7}\right)$

Sol. angular speed = angle × number of rev./s

$\omega = 2\pi \times \frac{14}{25} = 3.52$ rad/s

Ex.47 Earth revolves around the Sun in 365 days. Calculate its angular speed.

Sol. T = 365 days = 365 × 24 × 60 × 60 s

So, $\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60} = 1.99 \times 10^{-7}$ rad/s

Ex.48 A particle moves in a circle of radius 2 m and completes 5 revolutions in 10 seconds. Calculate the following:

(i) Angular velocity (ii) Linear velocity

Sol. Since, it completes 5 revolutions in 10 seconds.

$\therefore$ Time period = $\frac{10}{5} = 2$s

(i) Now angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

(ii) Linear velocity is given by

$v = r\omega = 2\pi$

$\therefore$ $v = 2\pi$ m/s

Ex.49 The length of second's needle in a watch is 1.2 cm. Calculate the following:

(i) Angular velocity and (ii) Linear velocity of the tip of the needle.

Sol. (i) We know that the second's needle in a watch completes one revolution in 60 seconds.

$\therefore$ Time period, T = 60 s

Angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

(ii) Length of the needle = 1.2 cm = Radius of the circle

Linear velocity of the tip of the needle is given by

$v = r\omega = 1.2 \times \frac{\pi}{30} = \frac{\pi}{25}$ **or** $v = \frac{\pi}{2s} = 1.266 \times 10^{-1}$ cm/sec.

Ex.50 Earth revolves around the sun in 365 days. Calculate its angular velocity.

Sol. Time period, T = 365 days = 365 × 24 × 60 × 60 seconds

Angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60}$ rad/s = 1.99×10^{-7} rad/s.

EXERCISE-1

➢ Very Short Answer Type Questions

Q.1 A ball is thrown up with a certain velocity. It attains a height of 40 m and comes back to the thrower. Find the distance and magnitude of displacement.

Q.2 What is the S.I. unit of displacement?

Q.3 A horse runs a distance of 1200 m in 3 min and 20 s. What is the speed of the horse?

Q.4 What is the S.I. unit of velocity?

Q.5 What is the S.I. unit of acceleration?

➢ Short Answer Type Questions – Type I

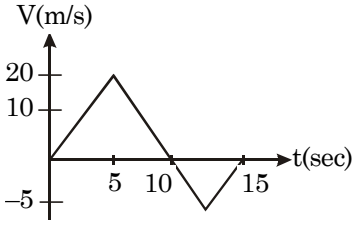
Q.6 Distance and displacement are equal in some cases. Give reasons.

Q.7 A stone is thrown upwards, reaches a height h and comes back. What are the distance moved and displacement?

Q.8 Under what condition(s) is the magnitude of average velocity of an object equal to its average speed?

Q.9 Define uniform circular motion.

Q.10 From the following (V-t) graph find:



(i) Distance and displacement in 10 second.

(ii) Distance and displacement in 15 second.

➢ Short Answer Type Questions – Type II

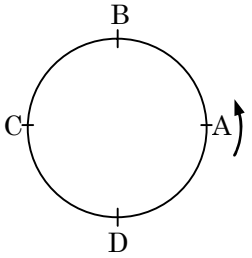
Q.11 Give three examples to explain that motion is relative.

Q.12 Which of the following is true for displacement?

(i) It cannot be zero.

(ii) Its magnitude is greater than the distance travelled by the object.

Q.13 A particle moves along a circle of radius R as shown in figure. It starts from A and moves in anti-clockwise direction.



Calculate the distance travelled and displacement:

(i) From A to B

(ii) From A to C

(iii) From A to D

Q.14 A train covers 80 km in 2 hours. Find its average speed in kmh^{-1}, m min^{-1} and ms^{-1}.

Q.15 An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed, if it takes 24 hours to revolve around the earth.

Q.16 Which one of the following have maximum and the least average speed?

(i) Sanjeev moving with 12 kmh^{-1}

(ii) Rajeev running with 5 ms^{-1}

(iii) Kabir moving with 150 m min^{-1}

Q.17 What do you mean by negative and positive acceleration? Explain.

Q.18 Why circular motion is accelerated motion?

Q.19 Draw the graph for uniform motion.
(i) Displacement - Time
(ii) Velocity - Time

Q.20 An engine is moving with a velocity 44 m/s. After applying the brakes, it stops after covering a distance of 121 m. Calculate retardation and time taken by the engine to stop.

➢ Long Answer Type Questions

Q.21 Define rest and motion and give two examples of each.

Q.22 Write five difference between distance and displacement.

Q.23 Write the short notes on:
(i) Uniform motion
(ii) Non uniform motion
(iii) Average speed
(iv) Velocity

Q.24 The distance between two points A and B is 100 m. A person moves from A to B with a speed of 20 m/s and from B to A with a speed of 25 m/s. Calculate average speed and average velocity.

Q.25 A body starting with initial velocity u moves with a constant acceleration a. Find the expression for distance travelled in nth seconds.

EXERCISE-2

Q.1 ABC is the shortest path length between the two points and ADC is the actual path length. Then which of the two corresponds to displacement?
(A) ADC (B) ABC
(C) Can't say (D) None of these

Q.2 Rest and motion both are
(A) Relative terms
(B) Absolute terms
(C) Can't say
(D) None of these

Q.3 An object has travelled 10 km in 15 minutes, its displacement will be
(A) 10 km
(B) zero
(C) More than 10 km
(D) Cannot be predicted

Q.4 Which of the following does not need direction to be defined completely
(A) Speed (B) Velocity
(C) Force (D) Displacement

Q.5 Speedometer is a device, which is used to measure
(A) Distance (B) Displacement
(C) Speed (D) None of these

Q.6 A boy travels 50km with 5km/hr and then for next 4hr travels with a uniform speed of 20km/hr. What is the average speed for the whole journey?
(A) 62/7km/hr (B) 65/7km/hr
(C) 60/7km/hr (D) 9km/hr

Q.7 Magnitude of average speed of an object is equal to its average velocity if
(A) it is moving in a definite direction.
(B) its initial and final positions are same.
(C) and only if it is in a uniform motion.
(D) they travel equal distances.

Q.8 From the top of a tower, a particle is projected upwards and it reaches the ground after 5 s. The initial velocity of the particle is 12 m/s, the height of the tower is
(A) 55 m (B) 65 m
(C) 75 m (D) 85 m

Q.9 A train passes over a 400 m long bridge. If the speed of the train is 30 m/s and the train takes 20 s to cross the bridge, then the length of the train is
(A) 400 m (B) 600 m
(C) 800 m (D) 200 m

Q.10 If a body covers a distance d with velocity v_1 and another distance d with same velocity v_2, then average velocity for the whole journey would be equal to
(A) $\frac{2v_1v_2}{v_1+v_2}$ (B) $\frac{v_1v_2}{v_1+v_2}$
(C) $\frac{v_1v_2}{2v_1+v_2}$ (D) $\frac{2(v_1v_2)}{v_1v_2}$

Q.11 If a body covers some distance with speed v_1 for time t_1 and some another distance with speed v_2 for some time t_2. Then what would be the average velocity for the whole duration?
(A) $\frac{v_1v_2}{2}$ (B) $\frac{v_1t_1+v_2t_2}{t_1+t_2}$
(C) $\frac{v_1v_2}{v_1+v_2}$ (D) $\frac{2v_1v_2}{v_1+v_2}$

Q.12 A body strikes the floor vertically with a speed 'u' and rebounds at the same speed. The change in velocity would be
(A) u (B) 3u (C) 2u (D) zero

Q.13 One car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is
(A) 40 km/hr (B) 80 km/hr
(C) $46\frac{2}{3}$ km/hr (D) 36 km/hr

Q.14 A 150 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 850 meters is

(A) 56 sec (B) 68 sec
(C) 80 sec (D) 92 sec

Q.15 The ratio of the numerical values of the average velocity and average speed of a body is always

(A) Unity (B) Unity or less
(C) Unity or more (D) Less than unity

Q.16 The formula for average velocity $\frac{u+v}{2}$ is valid for the case when acceleration is

(A) Variable (B) Zero
(C) Uniform (D) None of these

Q.17 If a body travels 20m in 10s starting from rest then the acceleration of the particle is:

(A) $0.4 m/s^2$ (B) $0.6 m/s^2$
(C) $1 m/s^2$ (D) $0.8 m/s^2$

Q.18 For the equation $s = ut - \frac{1}{2}at^2$ acceleration is in the:

(A) Opposite direction of initial velocity
(B) Opposite direction of displacement
(C) Same direction of initial velocity
(D) Both (A) and (B)

Q.19 An aeroplane lands at 432 km/h and stops after covering a runway of 4 km. The time in which it comes to rest is

(A) 1.8s (B) 60s
(C) 66.6s (D) 150s

Q.20. The correct statement from the following is

(A) A body having zero velocity will not necessarily have zero acceleration
(B) A body having zero velocity will necessarily have zero acceleration
(C) A body having uniform speed can have only uniform acceleration
(D) A body having non-uniform velocity will have zero acceleration

Q.21 The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is 0.34 ms^{-1}. If the change in velocity of the body is $0.18 ms^{-1}$ during this time, its uniform acceleration is

(A) $0.01 ms^{-2}$ (B) $0.02 ms^{-2}$
(C) $0.03 ms^{-2}$ (D) 0.04 ms

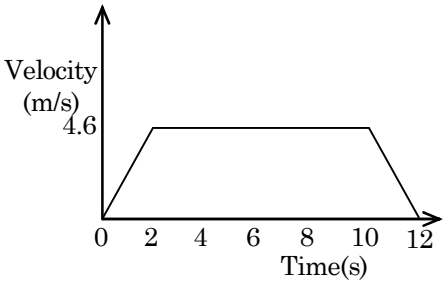
Q.22 A body under the action of several forces will have zero acceleration

(A) when the body is very light
(B) when the body is very heavy
(C) when the body is a point body
(D) when the vector sum of all the forces acting on it is zero

Q.23 The x and y coordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in metre and t in seconds. The acceleration of particle at t = 5s is

(A) Zero (B) $8 m/s^2$
(C) $20 m/s^2$ (D) $40 m/s^2$

Q.24 In the following velocity-time graph of a moving object

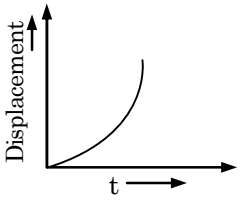


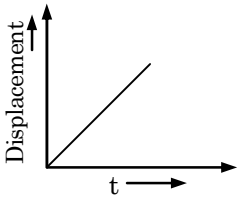
(A) Acceleration in the first 2 seconds is 2.3 ms^{-2}
(B) Acceleration in the last 2 seconds is $2.3 ms^{-2}$
(C) Motion is non-uniform between second and tenth second
(D) The body comes to rest at t = 2 and t =10 sec

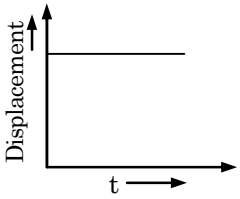
Q.25 The distance – time graph for an object moving with uniform speed is

(A) a straight line (B) is curvilinear
(C) rectilinear (D) parabolic

Q.26 Which of the following is correct for uniformly accelerated motion?

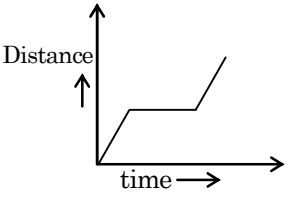
(A) 

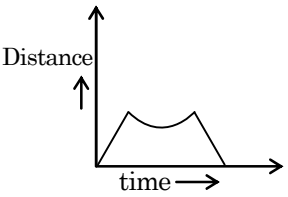
(B) 

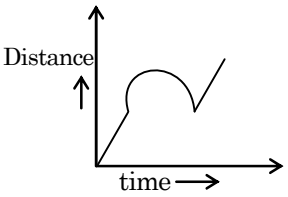
(C) 

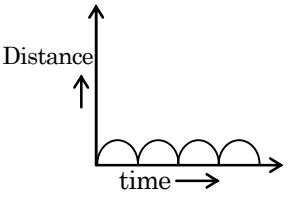
(D) All are correct

Q.27 Which of the following graph is possible?

(A) 

(B) 

(C) 

(D) 

Q.28 A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?

(A) 1.5 cm (B) 1.0 cm
(C) 3.0 cm (D) 2.0 cm

Q.29 A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(A) $S_1 = S_2$ (B) $S_1 = S_2/3$
(C) $S_1 = S_2/2$ (D) $S_1 = S_2/4$

Q.30 A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds with constant acceleration, then

(A) y = x (B) y = 2x
(C) y = 3x (D) y = 4x

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

Q.1 Value of one Fermi is

(A) 10^{-13}metre

(B) 10^{-14} metre

(C) 10^{-15} metre

(D) 10^{-16} metre

Q.2 A student starts with a velocity 40 km/hr for school at 4 km away from his house. Due to closing of school he returns soon to his house with a velocity of 60km/hr.His average velocity will be

(A) zero

(B) 10 km/hr

(C) 48 km/hr

(D) 50 km/hr

Q.3 A person takes time t to go once around a circular path of diameter 2R. The speed (v) of this person would be

(A) $\frac{t}{2\pi R}$ (B) $\frac{2\pi R}{t}$

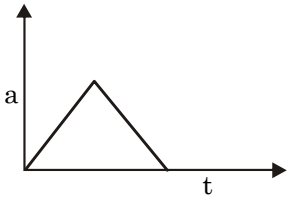
(C) $\frac{\pi R^2}{t}$ (D) $2\pi R.t$

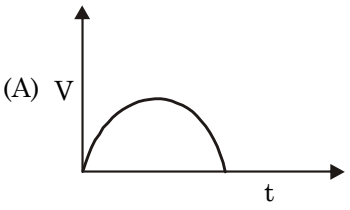
Q.4 A car travels 40 kms at an average speed of 80 km/h and then travels 40 kms at an average speed of 40 km/h. The average speed of the car for this 80 km trip is

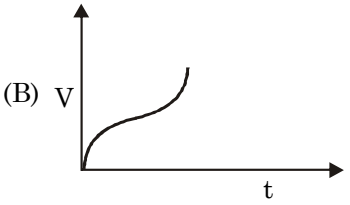
(A) 40 km/h (B) 45 km/h

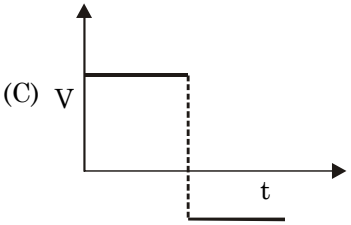
(C) 48 km/h (D) 53 km/h

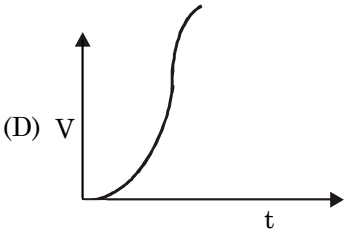
Q.5 The acceleration versus time graph of an object is as shown in figure. The corresponding velocity-time graph of the objects is



(A)


(B)


(C)


(D)


Q.6 A bullet of mass 10 g travelling horizontally with a velocity of 160 ms^{-1} strikes a stationary wooden block and comes to rest in 0.02 s. The distance of penetration of the bullet into the block will be

(A) 1.20 m (B) 1.60 m

(C) 2.00 m (D) 2.40 m

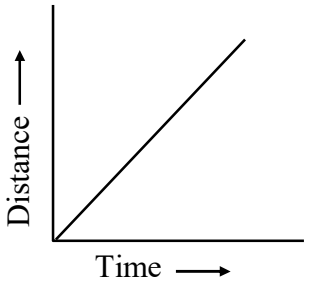
Q.7 The brakes applied to a car produce an acceleration of 8 m/s^2 in the opposite direction to the motion. If the car takes 3 seconds to stop after the application of brakes, the distance it travels during the time will be

(A) 30 m (B) 36 m

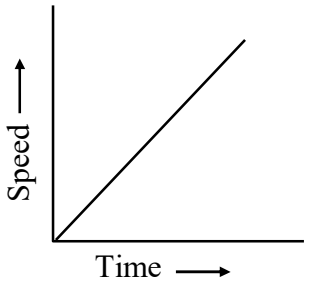
(C) 25 m (D) 40 m

Q.8 Consider the following five graphs (note the axes carefully). Which of the following represents motion at constant speed?

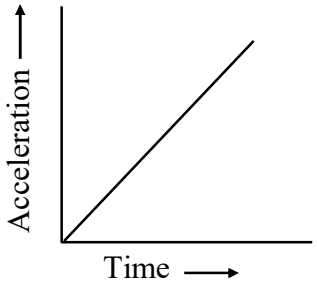
(1)



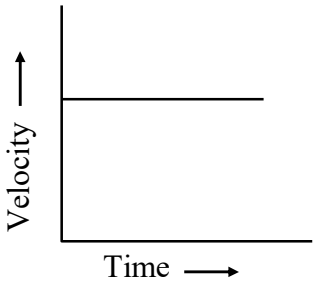
(2)



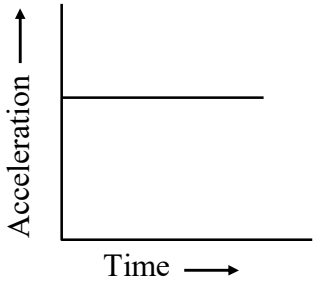
(3)



(4)

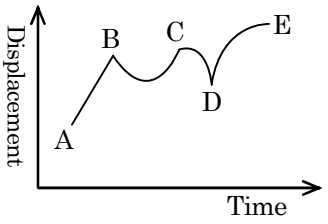


(5)



(A) 4 only (B) 4 and 5
(C) 1, 2 and 3 (D) 1 and 4

Q.9 The figure given below shows the displacement plotted time for a particle. In which regions is the force acting on the particle zero?



(A) AB (B) BC
(C) CD (D) DE

Q.10 Correct relation is.....
(A) $v^2 = u^2 + 2a^2s^2$
(B) $v^2 = u^2 - 2a^2s^2$
(C) $v^2 = u^2 + 2as$
(D) $v^2 = u^2 + 2a^2s$

Q.11 Two cars of unequal masses use similar tyres. If they are moving with same initial speed, the minimum stopping distance (air friction = 0)
(A) is smaller for the heavier car.
(B) is same for both the cars
(C) is smaller for the lighter car.
(D) depends on the volume of the car

Q.12 A ball hits a wall horizontally with a velocity of 6.0 ms^{-1}. After hitting wall it rebounds horizontally with a velocity of 4.4 ms^{-1}. If the balls remains in the contact of all for 0.040 sec. the acceleration of ball would be
(A) $-260\ m/s^2$ (B) $+260\ m/s^2$
(C) $-26\ m/s^2$ (D) $+26\ m/s^2$

Q.13 A man running with a uniform speed 'u' on a straight road observes a stationary bus at a distance 'd' ahead of him. At that instant, the bus starts with an acceleration 'a'. The condition that he would be able to catch the bus is

(A) $d \le \frac{u^2}{a}$ (B) $d \le \frac{u^2}{2a}$

(C) $d \le \frac{u^2}{3a}$ (D) $d \le \frac{u^2}{4a}$

Q.14 A car is moving with a constant speed of 70 km/h. Which of the following statements is correct?

(A) The acceleration of the car is definitely zero.

(B) The car has an acceleration only if it is moving along a curved path

(C) The car may have an acceleration even if it is moving along a straight path

(D) The car may not have an acceleration even if it is moving along a curved path

Q.15 A body cover half of the distance with a speed of 20m/s and the other half with 30m/s. The average speed of the body during the whole journey is

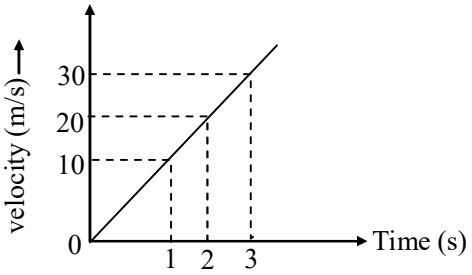
(A) Zero

(B) 24m/s

(C) 25 m/s

(D) None of the above

Q.16. Velocity-time graph of a body moving with uniform acceleration is shown in the diagram. The distance travelled by the body in 3 second is



(A) 90 m (B) 45 m

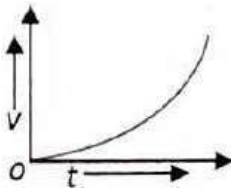
(C) zero (D) 10 m

Q.17 A body starts from rest is accelerated uniformly for 30s. If S1, S2, S3 are the distances travelled in first 10s; next 10s and last 10s respectively, then S1 : S2 :S3 is

(A) 1 : 2 : 3 (B) 1 : 1 : 1

(C) 1 : 3 : 5 (D) 1 : 3 : 9

Q.18 The velocity – time graph of a moving body is shown in the figure. Which of the following statements is true?



(A) The acceleration is constant and positive.

(B) The acceleration is constant and negative.

(C) The acceleration is increased and positive.

(D) The acceleration is decreasing and negative.

Q.19 If the distance travelled by an object is zero, then the displacement of the object is :

(A) zero

(B) not zero

(C) negative

(D) May or may not be zero

Q.20 How much time the satellite will take to complete one revolution around the earth, if velocity of satellite is 3.14 km/s and its height above earth's surface is 3600 km. (Radius of earth is 6400 km)

(A) 2000 S (B) 20000 S

(C) 1000 S (D) 10000 S

ANSWER KEY

EXERCISE - 1

1. 80 m and zero

3. 6m/s

7. 2h, 0

10. (i) 100 m, 100 m (ii) 112.5 m, 87.5 m

13. (i) $\frac{\pi R}{2}, \sqrt{2}R$ (ii) πR, $2R$ (iii) $\frac{3\pi R}{2}, \sqrt{2}R$

14. 40km/hr, 666.67m/min, 11.11m/sec

15. 3.069 km/s

20. 8 m/sec^2, 5.5s

24. 22.2 m/s

EXERCISE - 2

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	A	D	A	C	B	A	B	D	A	B	C	D	C	B
Ques.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	A	D	C	A	B	D	B	A	A	A	A	B	B	C

EXERCISE - 3

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	A	B	D	D	B	B	D	A	C	D	A	B	B	B
Ques.	**16**	**17**	**18**	**19**	**20**										
Ans.	B	C	A	A	B										

SOLUTIONS

EXERCISE-1

➢ Very Short Answer Type Questions

Sol.1 40 m.

Displacement = 0
Distance = 40m + 40m = 80m

Sol.2 Metre

Sol.3 $\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{1200}{200} = 6 \text{ m/sec.}$

Sol.4 m/sec

Sol.5 m/sec^2

➢ Short Answer Type Questions – Type I

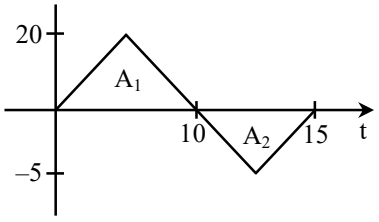
Sol.6 When body moving in straight line in certain direction.

Sol.7 Displacement = 0
Distance = 2h
Travelled

Sol.8 When body moving in straight line in certain direction.

Sol.9 If a body covers equal distance in equal interval along the circumference of the circle in equal interval of time.

Sol.10



Area $A_1 = \frac{1}{2} \times 20 \times 10 = 100$

Area $A_2 = -\frac{1}{2} \times 5 \times 5 = -12.5$

(i) Distance = Displacement = Area A_1
= 100 m.

(ii) Displacement = Area A_1 + Area A_2
= 100 – 12.5 = 87.5 m

Distance = (Area A_1) + (Area A_2)
= 100 + 12.5 = 112.5 m.

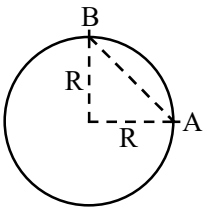
➢ Short Answer Type Questions – Type II

Sol.11 A train is moving on the track, the passenger are seated, will be stationary w.r.t. to each other but in moving condition w.r.t. to station.

Sol.12 Both are incorrect.

Sol.13 (i) $A \rightarrow B$

Distance $= \frac{1}{4}(2\pi R) = \frac{\pi}{2} R$



Displacement = $\sqrt{2}\,R$

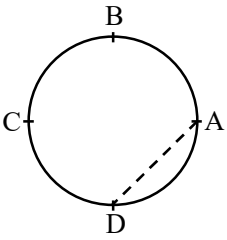
(ii) $A \rightarrow C$

Distance $= \frac{1}{2}(2\pi R) = \pi R$

Displacement = 2R

(iii) $A \rightarrow D$

Distance = $\frac{3}{4}(2\pi R) = \frac{3\pi R}{2}$



$\therefore$ Displacement = $\sqrt{2}\,R$

Sol.14 Avg. Speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{80\,\text{km}}{2\,\text{hr}} = 40$ Km/hr.

or $= \frac{80\times10^3\,\text{m}}{2\times60\,\text{min}} = 666.67\ \text{m/min}.$

or $= \frac{80\times10^3\,\text{m}}{2\times60\times60\,\text{sec}} = 11.12\ \text{m/min}.$

Sol.15 Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{2\pi R}{t} = \frac{2\pi R(42250)}{24}$

= 11,055.42 Km/hr

Sol.16 Convert all the units in SI (m/sec)

(i) 12 km/hr = 3.33 m/sec

(ii) 5 m/sec

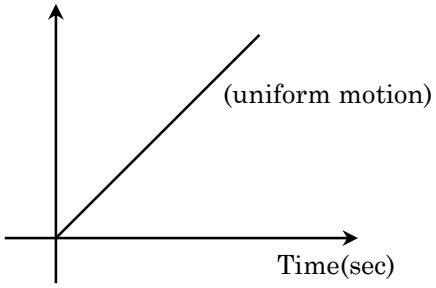
(iii) 150 m/min = 2.5 m/sec

So Rajeev has maximum speed & kabir has minimum speed.

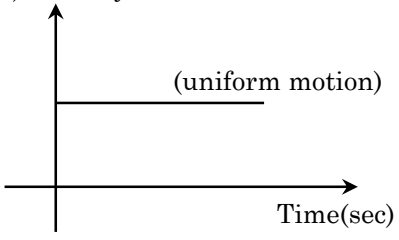
Sol.17 If direction of acceleration is in the direction of motion of body then it be positive acceleration and if the acceleration is in opposite direction of motion of body the it be negative acceleration.

Sol.18 In circular motion velocity is changing at every instant that's why circular motion is called as accelerated motion.

Sol.19 (i) Displacement(m)



(ii) Velocity



Sol.20 u = 44 m/sec

s = 121 M

v = 0

By using. Equation of motion.

$v^2 = u^2 + 2as$

$0 = (44)^2 + 2(a)\ (121)$

$a = -8\text{m/sec}^2$

$v = u + at$

$0 = 44 + (-8)\ (t)$

t = 5.5 sec

➢ Long Answer Type Questions

Sol.21 An object is said to be at rest if it does not change its position w.r.t. its surrounding with the passage of time.

A body is said to be in motion if its position change continuously with the surroundings with the passage of time.

Although rest it motion are relative terms.

Sol.22

Distance	Displacement
1. Distance is the length of the path actually traveled by a body in an direction.	1. Displacement is the short distance between the initial and the final position.
2. Distance b/w two given points depends upon the path chosen	2. Displacement b/w two points is measured by the straight path b/w the points.
3. Distance is always positive.	3. Displacement may be positive negative or zero.
4. Distance is a scalar quantity.	4. Displacement is vector quantity.
5. Distance will never decreases	5. Displacement my decreases.

Sol.23 (i) Uniform motion :

If an object moves equal distance in equal interval of time in a certain direction.

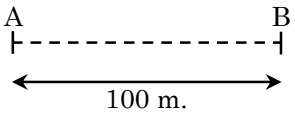
(ii) Non uniform motion :

If an object moves unequal distance in equal interval of time.

(iii) Average speed :

Total distance traveled by body divided by total time taken.

Sol.24



$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{100+100}{\frac{100}{20}+\frac{100}{25}} = \frac{200}{9} \text{ m/sec}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{total time}} = 0$$

Sol.25 $S_{n^{th}} = S_n - S_{n-1}$

$$= \left(ut + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right)$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{a}{2} + an$$

$$S_{n^{th}} = u - \frac{a}{2} + \frac{an}{2}$$

EXERCISE-2

Sol.1 Shortest path between two points is shown as displacement.

Sol.2 Rest and motion both are relative terms.

Sol.3 Displacement ≤ distance traveled.
So cannot be predicted by this information only.

Sol.4 Speed is a scalar quantity.

Sol.5 odometer is used to measure speed.

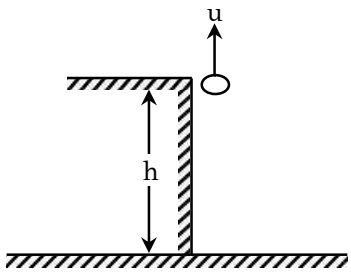
Sol.6 $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{S_1 + S_2}{t_1 + t_2}$$

$$= \frac{50 + (20 \times 4)}{\left(\frac{50}{5}\right) + 4} = \frac{65}{7} \text{ km/hr.}$$

Sol.7 |avg speed| = |avg velocity|
⇒ distance = displacement
It is only possible when body moving in definite direction.

Sol.8 U = + 12 m/sec
t = 5 sec
s = –h
a = –g = – 10 m/sec²



$$S = ut + \frac{1}{2}at^2$$

$$-h = 12 \times 5 + \frac{1}{2}(-10)(25)$$

h = 65 m

Sol.9 Let the length of the train is x.
Distance = Speed × Time
(400 + x) = 30 × 20
x = 200 m

Sol.10 $$\text{Average velocity} = \frac{\text{total displacement}}{\text{time taken}}$$

$$= \frac{2d}{t_1 + t_2}$$

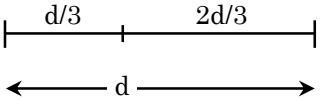
$$= \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

Sol.11 $$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

$$= \frac{v_1t_1 + v_2t_2}{t_1 + t_2}$$

Sol.12 Change in velocity
= final velocity – initial velocity
$= u\hat{j} - (-u\hat{j}) = 2u\hat{j}$
(as velocity is a vector quantity)

Sol.13



$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{d}{t_1 + t_2}$$

$$= \frac{d}{\frac{d/3}{20} + \frac{2d/3}{60}}$$

$$= \frac{d}{\frac{d}{60} + \frac{d}{90}} = 36 \text{ km/hr}$$

Sol.14 Total distance travelled by train
= 850 m + 190 m
Speed = 45 km/hr

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{1}{43}\text{hr} = 80 \text{ sec.}$$

Sol.15 $|\text{avg velocity}| \le |\text{avg speed}|$

Sol.16 If uniform acceleration.

$v^2 = u^2 + 2as$

$$v^2 - u^2 = 2\left(\frac{v-u}{t}\right)s$$

$$s = \frac{(v+u)}{2}t$$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

$$= \frac{\frac{(v+u)}{2}t}{t} = \frac{v+u}{2}$$

Sol.17 $S = ut + \frac{1}{2}at^2$

$20 = 0 + \frac{1}{2} \times a \times 100 \Rightarrow a = 0.4 \text{ m/sec}^2$

Sol.18 acceleration is negative
initial velocity is positive.
displacement is positive.
So acceleration is in opposite direction & both displacement in velocity.

Sol.19 u = 432 Km/hr = 120 m/sec
s = 4km = 400m
$v^2 = u^2 + 2as$
$o = (120)^2 + 2(a)\ (4000)$
$a = -1.8 \text{ m/sec}^2$
$v = u + at$
$o = 120 - 1.8\ \pi t$

$$t = \frac{120}{1.8} = 66.66 \text{ sec}$$

Sol.20 A body having zero velocity will not necessarily have zero acceleration

Sol.21 $\text{avg velocity} = \frac{\text{total displacement}}{\text{total time taken}}$

$$0.34 = \frac{3.06}{t}$$

$$t = \frac{3.06}{0.34} = 9$$

$$\text{acceleration} = \frac{v-4}{t} = \frac{0.18}{9} = 0.02 \text{ m/sec}^2$$

Sol.22 $\overrightarrow{F_{net}} = m\vec{a} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} \ldots..$
If vector sum of all forces is zero.
acceleration of body will be zero.

Sol.23 $x = 7t + 4t^2 \Rightarrow v_x = \frac{dx}{dt} = 7 + 8t \Rightarrow a_x = 8$

$$y = 5t \Rightarrow v_y = \frac{dy}{dt} = 5 \Rightarrow a_y = 0$$

$$\overrightarrow{a_{net}} = a_x\hat{i} + a_y\hat{i} = 8\hat{i}$$

Sol.24 Slope of velocity time graph provide the value of acceleration.

time	$0 \to 2$	$a = 2.3 \text{ m/sec}^2$
time	$2 \to 10$	$a = 0$ (uniform motion)
time	$10 \to 12$	$a = -2.3 \text{ m/sec}^2$

Sol.25 Slope of distance time graph provides the value of speed.
If uniform speed slope is constant.
$\Rightarrow$ Straight line.

Sol.26 Uniformly accelerated motion means velocity is increasing.
Slope of displacement v/s time graph provide velocity

Sol.27 Distance always increase with time it never decrease with time.

Sol.28 Let is assume initial speed of bullet be U after a displacement of 3cm its velocity becomes u/2.

$v^2 = u^2 + 2as$

$$\left(\frac{u}{2}\right)^2 = u^2 + 2(a)(3) \Rightarrow a = -\frac{u}{8}$$

as the resistance is constant, finally velocity becomes zero.

$v^2 = u^2 + 2as$

$0 = \left(\frac{u}{2}\right) + 2\left(-\frac{u}{8}\right)s$

s = 1cm

Sol.29 $S = ut + \frac{1}{2}at^2$

$S_{20} = 0 + \frac{1}{2} \times a \times (20)^2$

$S_{10} = 0 + \frac{1}{2} \times a(10)^2$

$S_1 = S_{10} - 0 = 50a$

$S_2 = S_{20} - S_{10} = 200a - 50a = 150a$

$\frac{S_2}{3} = S_1$

Sol.30 $S = 4t + \frac{1}{2}at^2$

$S_4 = 0 + \frac{1}{\alpha}a \times (4)^2 = 8a$

$S_2 = 0 + \frac{1}{\alpha}a \times (2)^2 = 2a$

$x = S_2 - 0 = 2a$

$y = S_4 - S_2 = 6a$

$y = 3x$

EXERCISE-3

Sol.1 1 fermi = 10^{-15} m.

Sol.2 avg velocity $= \frac{\text{displacement}}{\text{total time}}$

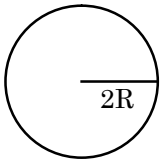
in this cost displacement = 0

$\Rightarrow$ avg velocity = 0

Sol.3 circumference = $2\pi(R)$

time = t

speed $= \frac{\text{distance travelled}}{\text{time}} = \frac{2\pi R}{t}$



Sol.4 average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

$= \frac{40+40}{t_1+t_2} = \frac{80}{\frac{40}{80}+\frac{40}{40}} = \frac{160}{3} \approx 53.33$ km/hr

Sol.5 Acceleration is increasing and then decreasing but acceleration is positive.

Velocity time graph first increases paraboliclly and then decreases in same manner.

Sol.6 u = 160 m/sec

t = 0.02 sec

v = u + at

o = 160 + a(0.02)

a = –8000 m/sec^2

$v^2 = 4^2 + 2as$

$o = (160)^2 + 2(-8000)(5)$

s = 1.6m

Sol.7 a = –8 m/sec^2

t = 3sec

v = 0

v = 4 + at

o = u + (–8)(3)

u = 24 m/sec

$S = 4 + \frac{1}{2}a + 2$

$= 24 \times 3 + \frac{1}{2} \times (-8)(3)^2 = 36$m.

Sol.8 Slope of distance v/s time graph provide speed.

Sol.9 A $\rightarrow$ B velocity is constant

$\Rightarrow$ Acceleration is zero.

$\Rightarrow$ Force is zero

Sol.10 $v^2 = u^2 + 2as$

Sol.11 $a = \frac{F}{m}$

Heavy car win have low retardation.

$\therefore v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$s = \frac{u^2}{2a}$

As retardation is law for heavy car distance cover will be more for it.

Sol.12 $\vec{a} = \frac{\overrightarrow{vf} - \overrightarrow{v_1}}{t} = \frac{-4.4\hat{i} - (6\hat{i})}{(0.04)} = -260$ m/sec^2

Sol.13 Let after time t man catches bus

For bus distance travel in Time t

$$s = \frac{1}{2}at^2$$

Man has to cover distance (d + s) in time t,

So

d + s = speed × time

$$d + \frac{1}{2}at^2 = u.t$$

or $\frac{2d}{a} + t^2 = \frac{2u}{a} \cdot t$

or $t^2 - \frac{2u}{a}t + \frac{2d}{a} = 0$

for positive roots.

$b^2 - 4ac \geq 0$

$$\frac{4u^2}{a^2} - 4(1)\left(\frac{2d}{a}\right) \geq 0$$

$$d \leq \frac{u^2}{2a}$$

Sol.14 In uniform circular motion speed is constant and it is an accelerated motion.

Sol.15 Let the total distance bed.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{d}{t_1 + t_2} = \frac{d}{\frac{d/2}{20} + \frac{d/2}{30}} = 24 \text{ m/sec.}$$

Sol.16 Slope of the line = (30-0)/3 = $10M/S^2$

a = 10m/s2

s = $\frac{1}{2}$ 10^3

s = 45m

Sol.17 $S_1 = \frac{1}{2}a(10)^2 = 100\left(\frac{a}{2}\right)$

$$S_2 - S_1 = \frac{1}{2}a(20)^2$$

$$S_3 - S_2 - S_1 = \frac{1}{2}a(30)^2$$

$$S_3 = 500\left(\frac{a}{2}\right)$$

$S_1 : S_2 : S_3 = 100 : 300 : 500 = 1 : 3 : 5$

Sol.18 Velocity of object is continue increasing parabolically hence acceleration is also increasing in positive manner.

Sol.19 If distance travel by object is zero its mean it is in the rest.

Sol.20 Radius of circle = 6400 + 3600 = 10,000 km

$$time = \frac{Distance}{Speed} = \frac{2 \times 3.14 \times 10,000}{3.14}$$

= 20,000 sec.

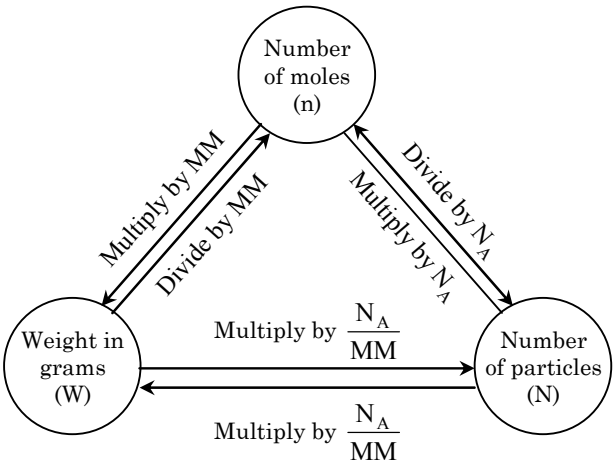
CHEMISTRY

Chapter

4 Atomic Foundations of Matter

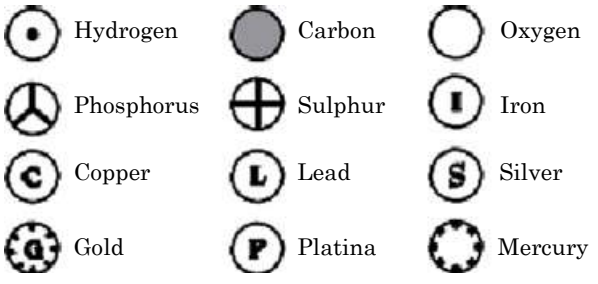
Chapter Outline

- Laws of Chemical Combination
- Dalton's Atomic Theory
- Atoms
- Symbols of an Element
- Molecules
- Concept of Mass
- Ions
- Chemical Formula
- Naming of Compounds
- Mole Concept
- Percentage Composition
- Empirical Formula
- Limiting Reagent

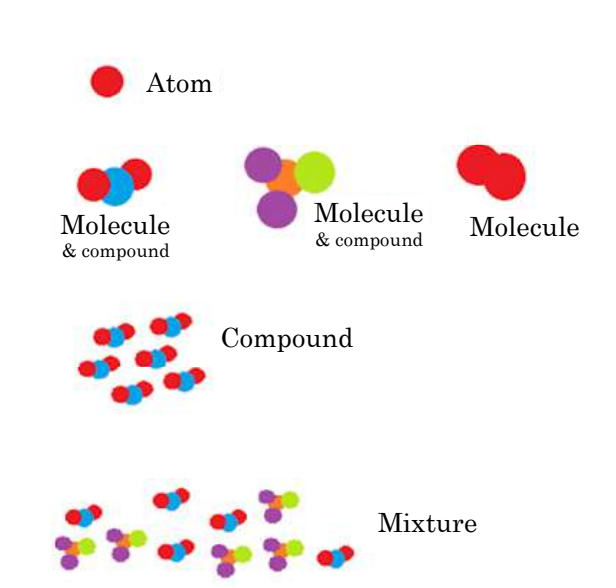


MM = Molar mass (GAM or GMM)

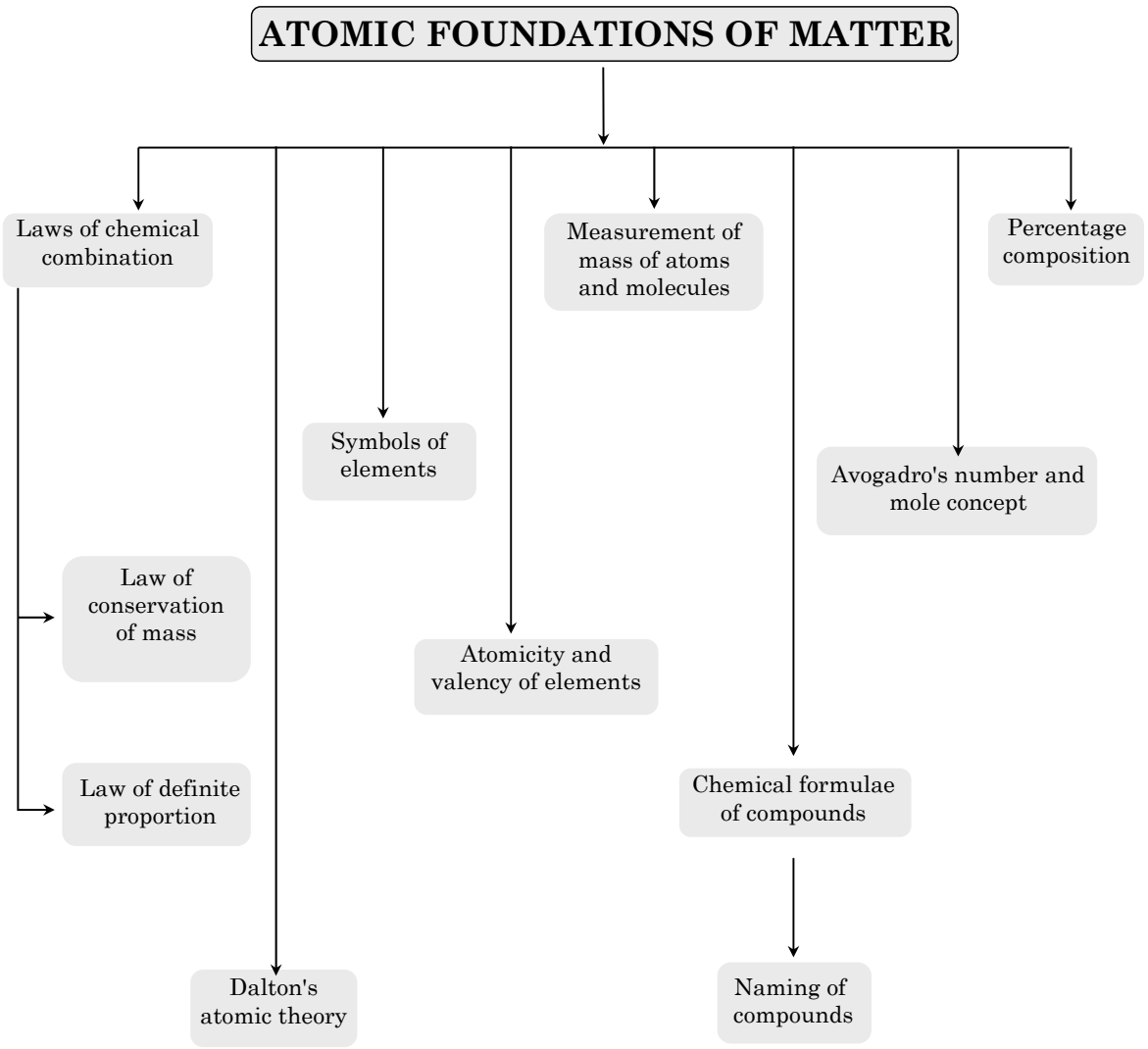
Mole



Symbols of an elements



MIND MAP


ATOMIC FOUNDATIONS OF MATTER
Laws of chemical combination
Law of conservation of mass
Law of definite proportion
Dalton's atomic theory
Symbols of elements
Atomicity and valency of elements
Measurement of mass of atoms and molecules
Chemical formulae of compounds
Naming of compounds
Avogadro's number and mole concept
Percentage composition

ATOMIC FOUNDATIONS OF MATTER

Introduction

An atom is the smallest part of an element that retains the properties of that element.

It was first suggested by an ancient Greek, named Democritus (the Greek word "atomos" means indivisible).

Democritus theorized that if you take an object and cut it in half again and again you would eventually end up with some particle which could not be further divided.

Note: *In the neutral atom the total number of protons in the nucleus is equal to the total number of electrons revolving around the nucleus.*

Laws of Chemical Combination

Chemistry is an experimental science and deals mainly with the reactions carried out time to time. In eighteenth century, chemists made several experiments and observed that all the chemical reactions always take place in accordance to certain laws. The experimental studies revealed the following laws.

◈ Law of Conservation of Mass (Lavoisier, 1798)

A french chemist *Antoine L. Lavoisier* (*the father of chemistry*) proposed a very significant law known as law of conservation of mass.

The law can be stated as follows: "The total mass of the products obtained during any physical or chemical change, is always equal to the total mass of reactants taken at the beginning of the change".

The matter is neither created nor destroyed during any physical or chemical change. However, it may change from one form to another.

- **Illustration of law of conservation of mass:**

(i) If 100 g of calcium carbonate ($CaCO_3$) is strongly heated, we get 56 g of calcium oxide (lime) and 44 g of carbon dioxide. The total mass of products (56 + 44 = 100 g) is exactly the same as the mass of calcium carbonate taken initially to carry out the reaction.

$$\underset{\substack{\text{Calcium carbonate}\\ 100\text{g}}}{CaCO_3} \xrightarrow{\text{Heat}} \underbrace{\underset{\substack{\text{Calcium oxide (lime)}\\ 56\text{g}}}{CaO} + \underset{\substack{\text{Carbon dioxide}\\ 44\text{g}}}{CO_2}}_{100\text{ g}}$$

(ii) 12 g of carbon (C) consumes exactly 32 g of oxygen (O_2) during combustion and forms exactly 44 g of carbon dioxide. The total mass of reactants in this case (12 + 32 = 44 g) is exactly the same as the mass of the product i.e., (CO_2) obtained.

$$\underbrace{\underset{\substack{\text{Carbon}\\ 12\text{g}}}{C} + \underset{\substack{\text{Oxygen}\\ 32\text{g}}}{O_2}}_{44\text{ g}} \longrightarrow \underset{\substack{\text{Carbon dioxide}\\ 44\text{g}}}{CO_2}$$

Ex.1 1.7 g of silver nitrate dissolved in 100 g of water is taken. 0.585 g of sodium chloride dissolved in 100 g of water is added to it. In chemical reaction 1.435 g of silver chloride and 0.85 g of sodium nitrate are formed. Justify that the data obey law of conservation of mass.

Sol. Total mass of reactants (initially) = mass of $AgNO_3$ + mass of NaCl

$= 1.7 + 0.585$

$= 2.285$ g

Total mass of products (finally) = mass of AgCl + mass of $NaNO_3$

$= 1.435 + 0.85$

$= 2.285$ g

Thus total mass before reaction = total mass after reaction = 2.285 g.

This confirms the law of conservation of mass.

Ex.2 8.4 g of sodium bicarbonate was added to 6.0 g acetic acid solution. Carbon dioxide gas was produced which was allowed to escape while the residue (of sodium acetate and water) was found to weigh 10.0 g. What is the mass of carbon dioxide which escaped out into the atmosphere, keeping in view the law of conservation of mass?

Sol. $NaHCO_3 + CH_3COOH \rightarrow CH_3COONa + H_2O + CO_2$

According to the law of conservation of mass.

Total mass before reaction = mass of $NaHCO_3$ + mass of CH_3COOH

8.4 g 6.0 g

Total mass after reaction = $\underbrace{\text{mass of } CH_3COONa + \text{mass of } H_2O}_{10\,g}$ + mass of CO_2

$\therefore$ Mass of $CO_2 = 14.4 - 10.0 = 4.4$ g

◈ Law of constant proportion or Law of Definite Proportion (Proust 1799)

A French chemist, *Proust* in 1799 proposed the law of constant proportion or Law of definite proportion. According to this law,

"A chemical compound, no matter from which source it is obtained, always contains the same elements combined together in the definite proportion by mass".

For example: water (H_2O) is comprised of two elements hydrogen and oxygen in the ratio 1: 8 by mass. Pure water obtained from any source, always contain hydrogen and oxygen in the same fixed ratio 1: 8 by mass.

Note: *To solve numerical problems of law of constant proportion, calculate the percentage of constituents in each case and prove that they are the same.*

Ex.3 The common salt can be obtained from two different sources. In one sample, the percentage of chlorine was found to be 60.75 %. In the second sample, 3.888 g of chlorine were present in 6.4 g of the salt. Show that these data are in accordance to the law of constant proportion.

Sol. The first sample of common salt contains 60.75 % chlorine.

$\because$ 6.4 g of salt contains chlorine = 3.888 g

$\therefore$ 100 g of salt will contain chlorine = $\frac{3.888 \times 100}{6.4} = 60.75$ g

Hence, the percentage of chlorine in second sample = 60.75 %

Since, both the samples of common salt contain the same percentage of chlorine, the given data are in accordance to the law of constant proportion.

Ex.4 1.08 g of copper wire was allowed to react with nitric acid. The resulting solution when dried and ignited gave 1.35 g of copper oxide. In another experiment 2.30 g of copper oxide was heated in presence of hydrogen yielding 1.84 g of copper. Show that the above data are in accordance with the law of constant proportion.

Sol. **Case I:**

$$\underset{1.08g}{Cu} \xrightarrow{HNO_3} Cu(NO_3)_2 \xrightarrow{\Delta} \underset{1.35g}{CuO}$$

$\because$ 1.35 g CuO contains 1.08 g Cu

$\therefore$ 100 g CuO contains (1.08 × 100) / 1.35 g Cu = 80 g Cu

$\therefore$ % of Cu in CuO = 80 ; % of O in CuO = 20

Case II:

$$\underset{2.30g}{CuO} + H_2 \rightarrow \underset{1.84g}{Cu} + H_2O$$

$\because$ 2.30 g CuO contains 1.84 g Cu

$\therefore$ 100 g CuO contains (1.84 × 100)/2.30 = 80 g Cu

$\therefore$ % of Cu in CuO = 80 ; % of O in CuO = 20

Since the percentage of Cu and O_2 in CuO in both the cases is same, the data are in accordance with the law of definite proportion.

Ex.5 1.4 g of calcium oxide obtained by heating limestone was found to contain 0.4 g of oxygen. Another sample of 3.5 g of calcium oxide obtained by direct combination of calcium and oxygen was found to contain
2.5 g of calcium. Show that the data are in accordance with the law of definite proportion.

Sol. **Case I:**

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$$

$\because$ 1.4 g CaO contains 0.4 g O_2

$\therefore$ 100 g CaO contains (0.4 × 100)/1.4 = 28.57 g O_2

$\therefore$ % of O in CaO = 28.57

$\therefore$ % of Ca in CaO = 100 – 28.57 = 71.43

Case II:

$$Ca + \frac{1}{2} O_2 \rightarrow CaO$$

$\because$ 3.5 g CaO contains 2.5 g Ca

$\therefore$ 100 g CaO contains (2.5 × 100) / 3.5 = 71.43

$\therefore$ % of Ca in CaO = 71.43

and % of O in CaO = 100 – 71.43 = 28.57

Thus, percentage of Ca and oxygen in CaO samples are same. This confirms the law of definite proportion.

COMPETITIVE LEVEL

(C) Law of Multiple Proportion: This law is given by Johan Dalton. "When two elements combine to form more than one compound then the different mass of one element which combine with a fixed mass of the other element bear a simple whole number ratio to one another".

e.g.:

(i) Carbon and oxygen when combine, can form two oxides that are CO (carbon monoxide). CO_2 (carbon dioxide). In CO, 12 g carbon combined with 16 g of oxygen. In CO_2, 12 g carbon combined with 32 g of oxygen. Thus, we can see the mass of oxygen which combine with a constant mass of carbon (12 g) bear simple ratio of 16: 32 or 1: 2.

(ii) Nitrogen and oxygen combine to form five oxides, which are: Nitrous oxides (N_2O), nitric oxide (NO), nitrogen trioxide (N_2O_3), nitrogen tetraoxide (N_2O_4) and nitrogen pentoxide (N_2O_5).

Weights of oxygen which combine with the fixed weight of nitrogen in these oxides are calculated as under:

Oxides	Ratio of weights of nitrogen and oxygen in each compound
N_2O	28 : 16 = 14 : 8
NO	14 : 16 = 14 : 16
N_2O_3	28 : 48 = 14 : 24
N_2O_4	28 : 64 = 14 : 32
N_2O_5	28 : 80 = 14 : 40

Number of parts by weight of oxygen which combine with 14 parts by weight of nitrogen from the above are 8, 16, 24, 32 and 40 respectively. Their ratio is 1: 2: 3: 4: 5, which is a simple ratio. Hence the law illustrated.

Ex.6 Carbon is found to form two oxides, which contain 42.8% and 27.27% of carbon respectively. Show that these figures illustrate the law of multiple proportions.

Sol. % of carbon in first oxide = 42.8

$\therefore$ % of oxygen in first oxide = 100 – 42.8 = 57.2

% of carbon in second oxide = 27.27

$\therefore$ % of oxygen in second oxide

= 100 – 27.27 = 72.73

For the first oxide -

Mass of oxygen in grams that combines with 42.8 g of carbon = 57.2

$\therefore$ Mass of oxygen that combines with 1 g of carbon = $\frac{57.2}{42.8} = 1.34\,g$

For the second oxide -

Mass of oxygen in grams that combines with 27.27 g of carbon = 72.73

$\therefore$ Mass of oxygen that combines with 1 g of carbon = $\frac{72.73}{27.27} = 2.68\,g$

Ratio between the masses of oxygen that combine with a fixed mass (1 g) of carbon in the two oxides = 1.34: 2.68 or 1: 2 which is a simple ratio. Hence, this illustrates the law of multiple proportion.

◈ Gay Lussac's Law of Gaseous Volumes

According to Gay Lussac's Law of combining volume.

"Whenever gases react chemically, they do so in volumes, which bear a simple ratio to each other and also to the products if they are also gaseous, provided the volumes are measured at same temperature and pressure for all the gases".

e.g.: Consider the reaction between nitrogen and hydrogen. The balanced equation is written as

$$\underset{(1\text{ vol})}{N_2} + \underset{(3\text{ vol})}{3H_2} \rightarrow \underset{(2\text{ vol})}{2NH_3}$$

The ratio of nitrogen gas that reacts with hydrogen gas forming ammonia gas is 1: 3: 2.

To illustrate 100 ml of nitrogen gas will react with 300 ml of hydrogen gas to give 200 ml of ammonia gas.

◈ Avogadro's Law

Avogadro's law states that. "'Equal volumes of all gases contain the same number of moles under same conditions of temperature and pressure".

Illustration

If Iℓ of hydrogen gas at STP contains say, 'n' molecules, then according to Avogadro law.

Iℓ of oxygen gas at STP has the same number of molecules. i.e., 'n' molecules.

Iℓ of nitrogen gas at STP also has the same number of molecules, i.e., 'n' molecules and

Iℓ of CO_2 at STP will also have the same number of molecules, i.e., 'n' molecules.

◆ Applications of Avogadro's Law:

- In the calculation of atomicity of elementary gases.

  e.g.: 3 volumes of hydrogen combine with 1 volume of nitrogen to form two volumes of ammonia.

  Hydrogen + Nitrogen $\longrightarrow$ Ammonia

  3 vol. 1 vol. 2 vol.

  Applying Avogadro's hypothesis

  Hydrogen + Nitrogen $\longrightarrow$ Ammonia

  3n molecules n molecules 2 n molecules

  or

  3 molecule 1 molecule 2 molecule

  $\frac{3}{2}$ molecules $\frac{1}{2}$ molecules 1 molecules

Thus1 molecule of ammonia contains $\frac{1}{2}$ molecule of nitrogen. But 1 molecule of ammonia contains 1 atom of nitrogen. Hence $\frac{1}{2}$ molecule of nitrogen = 1 atom of nitrogen or 1 molecules of nitrogen = 2 atoms of nitrogen i.e. atomicity of nitrogen = 2.

Dalton's Atomic Theory (1808)

Keeping in view the laws of chemical combinations and the work of Greek philosophers, a meaningful atomic theory was finally proposed by John Dalton in 1808. The basic postulates of Dalton's theory are as follows:

(i) Each element is composed of extremely small particles called atoms.

(ii) All atoms of a given element are identical i.e., atoms of a particular element are alike in all aspects.

(iii) Atoms of different elements possess completely different properties and differ in all aspects.

(iv) Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions.

(v) Atoms of element take part in chemical reaction to form molecules.

(vi) In a given compound, the relative number and kind of atoms are constant.

(vii) Atom of same type of elements combine in different ratio to form more than one type of compounds.

◈ Merits of Dalton's Atomic Theory

(i) Dalton's theory provides us a conceptual picture of matter. Atoms are the basic building blocks of matter. They are the smallest units of an element that can combine with other elements in a chemical reaction. In compounds, the atoms of two or more elements combine in definite arrangement.

(ii) Dalton's theory embodies several simple laws of chemical combination that were known at that time. Postulate (iv) indicates for the law of conservation of mass. Postulate (vi) indicates for the law of definite proportion.

(iii) Dalton's theory also explains the law of multiple proportion. Postulate (vii) indicates for the law of multiple proportion.

(iv) It explains that what makes an element to differ from others.

◈ Limitations of Dalton's Atomic Theory

(i) Distinction between atoms and molecules: According to Dalton, the smallest particle of an element as well as of compound was atom, however, he called 'compound atoms' to be smallest particle of compound. Later on Amedeo Avogadro used the term molecule for the 'compound atom' i.e., molecule is the smallest particle of compound.

(ii) It could not explain why do atoms combine to form a molecule.

(iii) It could not explain the nature of forces which hold the atoms and molecules in solid, liquid and gaseous state.

(iv) The theory fails to explain the existence of isotopes and isobars.

Atoms

- All the matter is made up of atoms. An atom is the smallest particle of an element that can take part in a chemical reaction. Atoms of most of the elements are very reactive and do not exist in the free state. They exist in combination with the atoms of the same element or another element.
- Atoms are very, very small in size. The size of an atom is indicated by its radius which is called 'atomic radius'. Atomic radius is measured in 'nanometres'. The symbol of a nanometre is nm.

1 nanometre = $\frac{1}{10^9}$ metre **or** 1 nm = $\frac{1}{10^9}$ m

or 1 nm = 10^{-9} m

Hydrogen atom is the smallest atom of all.

Note: *Atoms cannot be viewed by simple optical microscopes. However, through modern techniques such as scanning tunneling microscope it is possible to produce magnified images of surfaces of elements showing atoms.*

Ex.7 How do atoms exist?

Sol. The atoms of only a few elements called noble gases (such as Helium, Neon, Argon, Krypton, Xenon and Radon) are chemically unreactive and exist in the free state (as single atoms). Atoms of most of the elements are chemically very reactive and do not exist in the free state (as single atoms).

Atoms usually exist in two ways.

(i) In the form of molecules

(ii) In the form of ions.

When atoms form molecules or ions, they become stable (because while doing so they acquire the stable electronic configuration like noble gases).

Molecules

According to Avogadro, molecule is the smallest unit of a substance (element or compound) which can exist independently.

It is important to know that the atoms of the same element or different elements can join together to form molecules. On this basis, molecules are of two types:

(1) Molecules of elements (2) Molecules of compounds

◈ Molecules of Elements

The molecule which is made up of two or more atoms of the same element that are chemically bonded together is called molecule of an element.

e,g.: H_2, N_2, O_2, Cl_2, O_3, P_4, S_8 etc. are the molecules of elements. Molecules of helium (He), neon (Ne), argon (Ar) etc. are made up of single atoms. These are chemically unreactive elements.

The number of atoms present in a single molecule of a substance is called atomicity of that molecule or substance. The molecules of elements composed of the same type of atoms are called *homoatomic* molecules. They may be monoatomic, diatomic, triatomic or polyatomic.

(i) Monoatomic molecules: The molecules of noble gases like helium (He), neon (Ne), argon (Ar), krypton (Kr), xenon (Xe) and radon (Rn) are composed of one atom only. Hence, they are monoatomic in nature and exist independently in monoatomic state.

(ii) Diatomic molecules: Diatomic molecules of most of the gases formed by non-metal atoms consists of two atoms of same element held strongly together by attractive forces

e.g.: a molecule of oxygen consists of two oxygen atoms and is represented as O_2.

Examples of other diatomic molecules are nitrogen molecule (N_2), hydrogen molecule (H_2), chlorine molecule (Cl_2), fluorine molecule (F_2) etc. In each of the above examples subscript 2 indicates that there are two atoms.

(iii) Triatomic molecules: Triatomaic molecules consists of three atoms of same element held tightly together by attractive forces

e.g.: a molecule of ozone consists of three atoms of oxygen and is represented as O_3.

(iv) Polyatomic molecules: The polyatomic molecules of some non-metals consist of more than three atoms and are referred as polyatomic molecules

e.g.: phosphorus molecules (P_4), sulphur molecule (S_8) etc.

The molecules of metals and some other elements, such as carbon (e.g. C-60) consists of a very large and indefinite number of atoms bonded together.

◈ Molecules of Compounds

Molecules in which atoms of different elements are chemically bonded together are called molecules of compounds. Compounds in which atoms of different elements are chemically bonded together are called molecular or covalent compounds.

Water (H_2O), ammonia (NH_3), carbon dioxide (CO_2), methane (CH_4), hydrogen chloride (HCl), sulphuric acid (H_2SO_4) etc. are all molecules of compounds.

In all these compounds, atoms of different elements are chemically bonded together in the constant ratio by mass of elements.

- **Heteroatomic molecules:** The molecules of compounds compared of two or more atoms of different elements linked together in definite proportions are called *heteroatomic* molecules.

  For example, a molecule of water is composed of two atoms of hydrogen (H) and one atom of oxygen (O) therefore, a molecule of water is represented as H_2O.

  The heteroatomic molecules of compounds can also be classified as diatomic (NO), triatomic (H_2O) and polyatomic (H_2O_2) depending upon the total number of atoms present in them.

Note: *Every compound is a molecule, but every molecule is not a compound.*

Molecules of Some Compounds

Compound	Constituent Elements	Ratio by Mass	Formula of the Molecule	Atomicity
Hydrogen chloride	Hydrogen (H), Chlorine (Cl)	1: 35.5	HCl	2
Water	Hydrogen (H), Oxygen (O)	1 : 8	H_2O	3
Ammonia	Nitrogen (N), Hydrogen (H)	14 : 3	NH_3	4
Carbon monoxide	Carbon (C), Oxygen (O)	3 : 4	CO	2
Sulphur dioxide	Sulphur (S), Oxygen (O)	1 : 1	SO_2	3
Nitrous oxide	Nitrogen (N), Oxygen (O)	7 : 4	N_2O	3
Nitric oxide	Nitrogen (N), Oxygen (O)	7 : 8	NO	2

◈ Properties of Covalent Compounds

- Covalent compounds have weak intermolecular forces.
- Therefore, they melt and boil easily.
- Most covalent compounds are soft in nature.
- They do not conduct electricity because they do not contain free ions.
- Exception: Graphite conducts electricity.
- Most covalent compounds do not dissolve in water.
- They dissolve in solvents like benzene, ether and petrol.
- Covalent compounds may exist as solids, liquids or gases.
- Atoms share electrons to complete their outermost shell.

Ions

An ion is a positively or negatively charged atom (or group of atoms). An ion is formed by the loss or gain of electrons by an atom, so it contains an unequal number of electrons and protons.

e.g.: Sodium ion (Na^+), magnesium ion (Mg^{2+}), chloride ion (Cl^-), and oxide ion (O^{2-}) etc.

◈ Type of Ions

1. **On the basis of nature of charge:** There are two types of ions: cations and anions.

   - **Cation:** A positively charged ion is known as cation. A cation is formed by the loss of one or more electrons by an atom.

     e.g.: Sodium atom loses 1 electron to form a sodium ion, Na^+, which is cation:

     $$Na \xrightarrow{-1 \text{ electron}} Na^+$$

Sodium atom	Sodium ion (cation)
Protons = 11 (+ charge)	Protons = 11 (+charge)
Electrons = 11 (– charge)	Electrons = 10 (–charge)
Overall charge = 0	Overall charge = 1+

     e.g.: A magnesium atom loses 2 electrons to form magnesium ion, so magnesium ion bears 2 units of positive charge and it is represented as Mg^{2+}

     $$Mg \longrightarrow Mg^{2+} + 2e^-$$

     e.g.: An aluminium atom loses 3 electrons to form aluminium ion, so aluminium ion bears 3 units of positive charge and it is represented as Al^{3+}

     $$Al \longrightarrow Al^{3+} + 3e^-$$

   - **Anion:** A negatively charged ion is known as anion. An anion is formed by the gain of one or more electrons by an atom.

     e.g.:A chlorine atom gains 1 electron to form a chloride ion, Cl^-, which is an anion.

     $$Cl \xrightarrow{+1 \text{ electron}} Cl^-$$

Chlorine atom	Chlorine ion (anion)
Protons = 17 (+ charge)	Protons = 17 (+charge)
Electrons = 17 (– charge)	Electrons = 18 (–charge)
Overall charge = 0	Overall charge = 1–

     e.g.: A oxygen atom gains 2 electrons to form oxide ion, so oxide ion bears 2 units of negative charge and it is represented as O^{2-}

$$O + 2e^- \longrightarrow O^{2-}$$

e.g.: A nitrogen atom gains 3 electrons to form nitride ion, so nitride ion bears 3 units of negative charge and it is represented as N^{3-}

$$N + 3e^- \longrightarrow N^{3-}$$

Note:
- *The ions of all the non metal elements are anions.*
- *The ions of all the metal elements are cations.*
- *Size of a cation is always smaller and anion is always greater than that of the corresponding neutral atom.*

2. **On the basis of number of atoms:** There are two types of ions: monoatomic ions and polyatomic ions.
   - **Monoatomic ions:** These ion which are formed from single atom are called simple ions.

     e.g.: Na^+, K^+, Ca^{2+}, O^{2-}, F^- etc.
   - **Polyatomic ions:** Those ions which are formed from groups of joined atoms are called compound ions

     e.g.: NH_4^+, H_3O^+, OH^- etc.

Chemical Formulae

The chemical formula of a compound or an element represents the composition of a molecule of the compound or an element, in terms of the symbols of elements and the number of atoms of each element present in one molecule of the compound or an element.

◈ Formulae of Elements

The chemical formula of an element is a representation of the composition of its molecule in which symbol represents the element and subscript represents, how many atoms are present in one molecule.

e.g.: One molecule of hydrogen element contains two atoms of hydrogen, therefore, the formula of hydrogen is H_2. It should, however, be noted that 2H represents two separate atoms of hydrogen , while H_2 represents one molecule of hydrogen.

◈ Formulae of Compounds

The chemical formula of a compound is a representation of the composition of its molecule in which symbol represents, which elements are present and the subscript shows us how many atoms of each element are present in one molecule of a compound.

e.g.: One molecule of carbon dioxide contains 1 atom of carbon and 2 atoms of oxygen. Hence, the formula of carbon dioxide is CO_2.

(i) In the chemical formula of a compound, the elements present are denoted by their symbols and the number of atoms of each element are denoted by writing their number as subscripts to the symbols of the respective elements.

e.g.: Water is a compound whose one molecule is made up of 2 atoms of hydrogen and 1 atom of oxygen and hence, its chemical formula is H_2O.

(ii) While writing the formula of an ionic compound, the metal is written on the left hand side, while the non metal is written on the right hand side.

e.g.: Magnesium oxide is written as MgO , Sodium chloride is written as NaCl etc.

The name of the metal remains as such, but that of the non-metal is changed to have the ending 'ide'.

(iii) Molecular compounds, formed by the combination between two different non-metals, are written in such a way that the less electronegative element is written on the left hand side, while the more electronegative element is written on the right hand side. In naming molecular compounds, the name of the less electronegative non-metal is written as such but the name of the more electronegative element is changed to have the ending 'ide'.

e.g.: H_2S is named as hydrogen sulphide and HCl is named as hydrogen chloride .

(iv) When there are more than one atoms of an element present in the formula of the compound, then the number of atoms are indicated by the use of appropriate prefixes (mono for 1, di for 2, tri for 3, tetra for 4 atoms etc. respectively) in the name of the compound.

e.g.: CO is named as carbon monoxide, CO_2 is named as carbon dioxide and CCl_4 is named as carbon tetrachloride.

(v) The prefixes are needed in naming those binary compounds in which the two non-metals form more than one compounds (by having different number of atoms).

e.g.: Two non-metals, nitrogen and oxygen, combine to form different compounds like nitrogen monoxide (NO), nitrogen dioxide (NO_2), dinitrogen trioxide (N_2O_3) etc.

(vi) If two non-metals form only one compound, then prefixes are not used in naming such compounds.

e.g.: Hydrogen and sulphur combine to form only one compound H_2S. So, H_2S is named as hydrogen sulphide and not as hydrogen monosulphide or dihydrogen monosulphide.

The number ratio of atoms present in a molecule can be determined by dividing the ratio by mass with the respective atomic masses of the corresponding elements and finding the simplest ratio.

LIST OF COMMON ELECTROVALENT POSITIVE RADICALS

Monovalent Electropositive		Bivalent Electropositive		Trivalent Electropositive		Tretravalent Electropositive	
1	Hydrogen H^+	1	Magnesium Mg^{2+}	1	Aluminium Al^{3+}	1	Stannic [Tin(IV)] Sn^{4+}
2	Ammonium NH_4^+	2	Calcium Ca^{2+}	2	Ferric [Iron(III)] Fe^{3+}	2	Plumbic [lead(IV)] Pb^{4+}
3	Sodium Na^+	3	Zinc Zn^{2+}	3	Chromium Cr^{3+}		
4	Potassium K^+	4	Plumbous [Lead(II)] Pb^{2+}				
5	Cuprous [Copper(I)] Cu^+	5	Cupric [Copper(II)] Cu^{2+}				
6	Argentous [Silver(I)] Ag^+	6	Argentic [Silver(II)] Ag^{2+}				
7	Mercurous [Mercury(I)] Hg^{1+}	7	Stannous [Tin(II)] Sn^{2+}				
		8	Ferrous [Iron(II)] Fe^{2+}				
		9	Mercuric [Mercury(II)] Hg^{2+}				
		10	Barium Ba^{2+}				

LIST OF COMMON ELECTROVALENT NEGATIVE RADICALS							
Monovalent Electronegative		**Bivalent Electronegative**		**Trivalent Electronegative**		**Tretravalent Electronegative**	
1	Fluoride F^-	1	Sulphate SO_4^{2-}	1	Nitride N^{3-}	1	Carbon C^{4-}
2	Chloride Cl^-	2	Sulphite SO_3^{2-}	2	Phosphide P^{3-}		
3	Bromide Br^-	3	Sulphide S^{2-}	3	Phosphite PO_3^{3-}		
4	Iodide I^-	4	Thiosulphate $S_2O_3^{2-}$	4	Phosphate PO_4^{3-}		
5	Hydride H^-	5	Zincate ZnO_2^{2-}				
6	Hydroxide OH^-	6	Oxide O^{2-}				
7	Nitrite NO_2^-	7	Peroxide O_2^{2-}				
8	Nitrate NO_3^-	8	Dichromate $Cr_2O_7^{2-}$				
9	Bicarbonate or Hydrogen carbonate HCO_3^-	9	Carbonate CO_3^{2-}				
10	Bisulphite or Hydreogen sulphite HSO_3^-	10	Silicate SiO_3^{2-}				
11	Bisulphide or Hydrogen sulphide HS^-						
12	Bisulphate or Hydrogen sulphate HSO_4^-						
13	Acetate CH_3COO^-						

◈ Ion and method of writing the chemical formula of an ionic compound

An ion is an atom or a group of atoms (of the same or different elements) that carries a positive or a negative charge and behaves like a single unit.

The species containing a single atom and carrying a positive or negative charge are called simple ions i.e., Na^+, Ca^{2+}, H^+, Cl^-, Br^- etc. The species containing a group of atoms and having a net charge on them are called polyatomic ions, i.e; NH_4^+, SO_4^{2-}, PO_4^{3-}, NO_3^- etc.

Positively charged ions are termed as cations, whereas the negatively charged ions are termed as anions.

The valency of an ion in usually equal to the number of charges present on it. Depending upon their valencies, ions are also referred to as monovalent (valency = 1), bivalent (valency = 2), trivalent (valency = 3) and tetravalent (valency = 4).

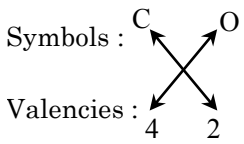
Note: *1. 2^+ means two positive charges, i.e., + + e.g. Ca^{2+} ; 3^+ means three positive charges, i.e., + + + e.g. Fe^{3+}; whereas 4^+ indicates the presence of four positive charges (+ + + +) e.g. Similarly 1^- means one negative charge i.e. –1 e.g. Cl^- & so on.*

2. Roman number written in brackets represent the valency of the ion i.e. Fe^{2+} as Fe(II) in compounds.

Ex.8 Writing the formula of carbon dioxide.

Sol. Following steps are used to write the formula of carbon dioxide -

- Carbon dioxide is a compound composed of two elements, carbon and oxygen. So, we first write their symbols C and O respectively.
- The valency of carbon is 4 and the valency of oxygen is 2. Now, these valencies are to be written under the corresponding symbols of elements.



- Now, the valencies of carbon and oxygen are to be exchanged. So, the subscript corresponding to C is 2 and that corresponding to O is 4. Hence, the formula of the compound becomes C_2O_4.
- But, the valencies 2 and 4 have a common factor 2. So, on dividing the whole formula by 2, we get the simplest formula CO_2. Thus, the formula of carbon dioxide is CO_2.

Ex.9 Write the formula of hydrogen sulphide.

Sol. Elements present in hydrogen sulphide are hydrogen and sulphur. Valency of hydrogen is 1 and that of sulphur is 2.

Symbol: H S H S H_2S_1 or H_2S

1 2 (Formula)

Cross over Valenceies

Valency: 1 2

In the formula H_2S:

Valency of H is 1, atoms of H are 2.

Valency of S is 2, atom of S is 1.

Total valencies of H-atom = $1 \times 2 = 2$

Total valency of S-atom = $2 \times 1 = 2$

Since total valency of atoms of metals and non-metals comes out to be equal, the formula (H_2S) is correct of hydrogen sulphide.

◈ Chemical Formula of Ionic Compound

The compounds which are made up of ions are known as ionic compounds. In an ionic compound, the positively charged ions (cations) and negatively charged ions (anions) are held together by the strong electrostatic forces of attraction. The forces which hold the ions together in an ionic compound are known as ionic bonds or electrovalent bonds. Since an ionic compound consists of an equal number of positive ions and negative ions, so the overall charge on an ionic compound is zero.

e.g.: Sodium chloride (NaCl) is an ionic compound which is made up of equal number of positively charged sodium ions (Na^+) and negatively charged chloride ions (Cl^-).

Some ionic compound and their ions

S. No.	Name	Formula	Ions present
1	Sodium chloride	NaCl	Na^+ and Cl^-
2	Potassium chloride	KCl	K^+ and Cl^-
3	Ammonium chloride	NH_4Cl	NH_4^+ and Cl^-
4	Magnesium chloride	$MgCl_2$	Mg^{2+} and Cl^-
5	Calcium chloride	$CaCl_2$	Ca^{2+} and Cl^-
6	Magnesium oxide	MgO	Mg^{2+} and O^{2-}
7	Calcium oxide	CaO	Ca^{2+} and O^{2-}
8	Aluminium oxide	Al_2O_3	Al^{3+} and O^{2-}
9	Sodium hydroxide	NaOH	Na^+ and OH^-
10	Copper sulphate	$CuSO_4$	Cu^{2+} and SO_4^{2-}
11	Calcium nitrate	$Ca(NO_3)_2$	Ca^{2+} and NO_3^-

An ionic compound is electrically neutral and is usually formed by the combination of positive and negative ions in such a way that the positive charge of the cation is completely neutralized by the negative charge present on the anion. This fact should be kept in mind while writing the formula of a compound.

The method used to write the formula of a compound is called *Criss-cross method*. It is carried out in the following steps:

(i) Write the symbols of cation and anion side by side. Say for example magnesium oxide. The cation Mg^{2+} i.e., the ion having positive charge is written first i.e., O^{2-}. The anion (O^{2-}) is written after it.

(ii) Write the charge of each ion on the top of its symbol. While doing so, ignore the + and – sign i.e. $\overset{2}{Mg}\ \ \overset{2}{O}$

(iii) Divide the charges by their highest common factor (if any) to get a simple ratio (in this case by 2).

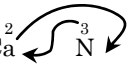
(iv) Interchange the charge and write them on the lower right side of the symbols i.e. $MgO(Mg_1O_1)$. The number 1 does not need to be written. This is known as (cris cross method).

(v) The formula of magnesium oxide MgO.

Let us consider another example of calcium nitride.

(i) Writing the symbols of cation and anion $Ca^{2+}\ N^{3-}$

(ii) Writing the charge of ions: $\overset{2}{Ca}\ \ \overset{3}{N}$

(iii) Interchange their charge: 

(iv) Thus, the formula of calcium nitride is Ca_3N_2.

◈ Properties of Ionic Compounds

- Ionic compounds are generally hard solids.
- They have a crystalline structure.
- Strong electrostatic force of attraction exists between ions.
- Therefore, a large amount of heat is required to break the bond.
- Ionic compounds conduct electricity only when dissolved in water or in molten form because ions become free to move.
- They do not conduct electricity in solid state.
- Most ionic compounds dissolve easily in water. Ex: Sodium chloride (NaCl).
- They are generally insoluble in petrol, kerosene and benzene.
- Positive and negative ions are strongly attracted to each other.
- Ionic compounds break easily when pressure is applied.

Naming Compounds from Their Formulae

◈ Naming Simple Compounds Formed by a Metal and a Non-metal

The name of a simple compound formed by a metal and a non-metal (or group of non-metals) can easily be written by identifying cation and anion in its formula. The name of the compound is obtained by first writing the name of the metal cation followed by the name of the non metal anion.

Formula	Name
NaCl	Sodium chloride
KCl	Potassium chloride
$MgCl_2$	Magnesium chloride
$CaCl_2$	Calcium chloride
MgO	Magnesium oxide
CaO	Calcium oxide
Al_2O_3	Aluminium oxide

Note: *The valency of an element is always a whole number. Some elements exhibit more than one valency, i.e., they have variable valency.*

Generally, the Latin / Greek name for the element is modified to end in 'ous' for the lower valency and to end in 'ic' for the higher valency.

The table below lists some elements that exhibit variable valencies.

Element	Symbol	Valency	Ions	Nomenclature
Copper	Cu	1 2	Cu^{1+} Cu^{2+}	Copper [I] or Cuprous Copper [II] or Cupric
Silver	Ag	1 2	Ag^{1+} Ag^{2+}	Silver [I] or Argentous Silver [II] or Argentic
Mercury	Hg	1 2	Hg^{1+} Hg^{2+}	Mercury [I] or Mercurous Mercury [II] or Mercuric
Iron	Fe	2 3	Fe^{2+} Fe^{3+}	Iron [II] or Ferrous Iron [III] or Ferric
Tin	Sn	2 4	Sn^{2+} Sn^{4+}	Tin [II] or Stannous Tin [IV] or Stannic
Lead	Pb	2 4	Pb^{2+} Pb^{4+}	Lead [II] or Plumbous Lead [IV] or Plumbic

In case of the metal forming the compound is capable of showing variable valencies, the valency of the metal is written in roman numerals in paranethessis after the name of the metal. This representation is called stock notation. For example:

Naming Simple Compounds from their formulae

Formula	Name
CuCl	Copper (I) chloride or Cuprous chloride
$CuCl_2$	Copper (II) chloride or Cupric chloride
$FeCl_2$	Ferrum (II) chloride or Ferrous chloride
$FeCl_3$	Ferrum (III) chloride or Ferric chloride

◈ Naming Compounds formed by Two non-metals

Compounds formed by two non-metals are named by writing first non metal and then other using the suffixes di, tri, tetra, penta etc. before the name of the non-metals if they appear with more than one atom. The name of more electronegative non metal comes in last. For example.

Formula	Name
CO_2	Caron dioxide
NO_2	Nitrogen dioxide
OF_2	Oxygen difluoride
PCl_3	Phoshorus trichloride
CCl_4	Carbon tetrachloride
PCl_5	Phsophorus pentachloride
N_2O_5	Dinitrogen pentoxide

Concept of Mass

Atoms of all the elements (except noble gases) are not able to exist independently. They combine with similar or different atomic species to form molecules or ions. These molecules or ions aggregate together in large numbers to form the matter which is around us.

Only a few elements, (noble gases) e.g.: helium, neon, argon, krypton, xenon and radon are able to exist freely in atomic state i.e., these shows independent existence and exist as mono atomic gases in nature.

The absolute atomic mass of an atom of hydrogen is 1 u or 1.67×10^{-24} g as well as absolute atomic masses of other atoms are also of the same order. Since the absolute atomic masses of all atoms are extremely small and are in the order of 10^{-24} g and therefore, it is very inconvenient to use mass of one atom in routine calculations. A new term *relative atomic mass* was therefore used. The relative atomic mass is its absolute mass compared to the absolute mass of an atom of a standard substance i.e. carbon, Thus,

$$\textit{Relative atomic mass} = \frac{\text{Absolute mass of atom of the given substance}}{\text{Absolute mass of an atom of a standard substance}}$$

Direct determination of relative mass as defined above is again impossible as it will again require absolute masses of the atoms.

The relative atomic mass of an element is the number which tells how many times an atom of that element is heavier than an atom of hydrogen.

◈ Definition of Mass

Mass is the measure of quantity of matter within a sample.

◈ Atomic Mass Unit

The atomic mass unit (amu) is equal to one-twelfth (1/12) of the mass of an atom of carbon-12.The mass of an atom of carbon-12 isotope was given the atomic mass of 12 units, i.e. 12 amu or 12 u.

The atomic masses of all other elements are now expressed in atomic mass units.

◈ Atomic Mass

- **Relative Atomic Mass:** The atomic mass of an element is a relative quantity and it is the mass of one atom of the element relative to one -twelfth (1/12) of the mass of one carbon-12 atom. Thus,

  $$\text{Relative atomic mass} = \frac{\text{Mass of one atom of the element}}{\frac{1}{12} \times \text{mass of one C} - 12 \text{ atom}}$$

  [1/12 the mass of one C-12 atom = 1 amu,

  1 amu = 1.66×10^{-24} g = 1.66×10^{-27} kg.]

- **Gram Atomic Mass:** The atomic mass of an element expressed in grams is called the Gram Atomic Mass of the element.

  $$\text{The number of gram -atoms} = \frac{\text{Mass of the element in grams}}{\text{Gram Atomic mass of the element}}$$

Note: *Most of the atomic masses determined with hydrogen as the standard were not whole numbers. Later on, O^{16} isotope of oxygen was taken as the reference substance and the relative atomic masses of most of the elements were calculated on its basis and found very close to whole numbers.*

In 1961, a new standard carbon-12, (C^{12}, the most stable isotope of carbon) was adopted by International Union of Chemistry to calculate the relative atomic masses of elements. The new scale adopted is called atomic mass unit scale.

◈ Molecular Mass

As the term itself explains, molecular mass is the mass of one molecule of a substance. But like an atom, a molecule is so small that it is impossible to determine its absolute mass. Therefore its better to refer the relative molecular mass.

- **Relative molecular mass:** The relative molecular mass of a substance is the mass of a molecule of the substance as compared to one-twelfth of the mass of one carbon -12 atom i.e.,

$$\text{Relative molecular mass} = \frac{\text{Mass of one molecule of the substance}}{\frac{1}{12} \times \text{mass of one C} - 12 \text{ atom}}$$

The molecular mass of a molecule, thus, represents the number of times it is heavier than 1/12 of the mass of an atom of carbon-12 isotope.

- **Gram Molecular Mass:** The molecular mass of a substance expressed in grams is called the Gram Molecular Mass of the substance.

$$\text{The number of gram molecules} = \frac{\text{Mass of the substance in grams}}{\text{Gram molecular mass of the substance}}$$

e.g.:

(i) Molecular mass of hydrogen (H_2) = 2u.

$\therefore$ Gram Molecular Mass of hydrogen (H_2) = 2 g .

(ii) Molecular mass of methane (CH_4) = 16u

$\therefore$ Gram Molecular Mass of methane (CH_4) = 16 g.

Note: *The molecular mass of a substance is the sum of the atomic masses of its constituent atoms present in a molecule.*

Ex.10 Calculate the molecular mass of water.

(Atomic masses: H = 1u, O = 16u).

Sol. The molecular formula of water is H_2O.

$\therefore$ Molecular mass of water = (2 × atomic mass of H) + (1 × atomic mass of O) = 2 × 1 + 1 × 16 = 18

i.e., molecular mass of water = 18 amu.

Ex.11 Find out the molecular mass of sulphuric acid. (Atomic mass: H = 1u, O = 16u, S = 32u).

Sol. The molecular formula of sulphuric acid is H_2SO_4

$\therefore$ Molecular mass of H_2SO_4 = (2 × atomic mass of H) + (1 × atomic mass of S) + (4 × atomic mass of O)

$= (2 \times 1) + (1 \times 32) + (4 \times 16) = 2 + 32 + 64 = 98$

i.e., Molecular mass of H_2SO_4 = 98 amu.

◈ Formula Mass

The term 'formula mass' is used for ionic compounds and others where discrete molecules do not exist, e.g.: sodium chloride, which is best represented as $(Na^+Cl^-)_n$, but for reasons of simplicity as NaCl or Na^+Cl^-. Here, formula mass means the sum of the masses of all the species in the formula.

Thus, the formula mass of sodium chloride = (atomic mass of sodium) + (atomic mass of chlorine)

$= 23 + 35.5$

$= 58.5$ amu

COMPETITIVE LEVEL

Mole Concept

Mole in Latin means heap or collection or pile. A mole of atoms is a collection of atoms whose total mass is the number of grams equal to the atomic mass in magnitude. Since an equal number of moles of different elements contain an equal number of atoms, it becomes convenient to express the amounts of the elements in terms of moles. A mole represents a definite number of particles, viz, atoms, molecules, ions or electrons. This definite number is called the Avogadro Number (now called the Avogadro constant) which is equal to 6.023×10^{23}.

A mole is defined as the amount of a substance that contains as many atoms, molecules, ions, electrons or other elementary particles as there are atoms in exactly 12 g of carbon -12 (^{12}C).

- **Moles of Atoms:**

  (i) 1 mole atoms of any element occupy a mass which is equal to the Gram Atomic Mass of that element.

  e.g.: 1 Mole of oxygen atoms weigh equal to Gram Atomic Mass of oxygen, i.e. 16 grams.

  (ii) The symbol of an element represents 6.023 x 10^{23} atoms (1 mole of atoms) of that element.

  e.g.: Symbol N represents 1 mole of nitrogen atoms and 2N represents 2 moles of nitrogen atoms.

- **Moles of Molecules:**

  (i) 1 mole molecules of any substance occupy a mass which is equal to the Gram Molecular Mass of that substance.

  e.g.: 1 mole of water (H_2O) molecules weigh equal to Gram Molecular Mass of water (H_2O), i.e. 18 grams.

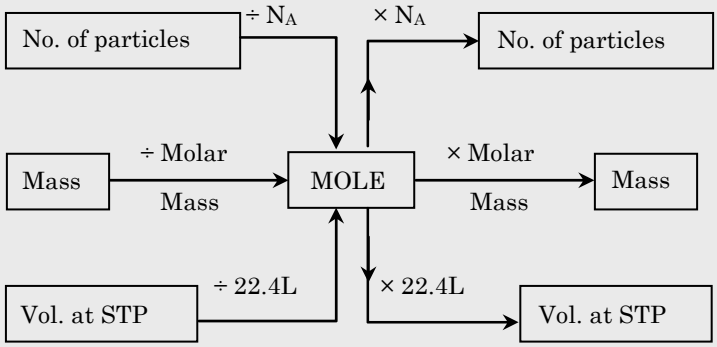
  (ii) The symbol of a compound represents 6.023 x 10^{23} molecules (1 mole of molecules) of that compound.

  e.g.: Symbol H_2O represents 1 mole of water molecules and 2 H_2O represents 2 moles of water molecules.

◈ Some important relations & Formulae

- 1 mole of atoms = Gram Atomic mass = mass of 6.023×10^{23} atoms
- 1 mole of molecules = Gram Molecular Mass = 6.023×10^{23} molecules
- Number of moles of atoms $= \dfrac{\text{Mass of element in grams}}{\text{Gram Atomic Mass of element}}$
- Number of moles of molecules $= \dfrac{\text{Mass of substance in grams}}{\text{Gram Molecular Mass of substance}}$
- Number of moles of molecules $= \dfrac{\text{No.of molecules of element}}{\text{Avogadro number}} = \dfrac{N}{N_A}$

- Number of moles of atoms $= \frac{\text{No. of atoms of an element}}{\text{Avogadro number}} = \frac{N}{N_A}$
- Number of atoms = No. of molecules × atomicity



Ex.12 To calculate the number of moles in 16 grams of Sulphur (Atomic mass of Sulphur = 32 u).

Sol. 1 mole of atoms = Gram Atomic Mass.

So, 1 mole of Sulphur atoms = Gram Atomic Mass of Sulphur = 32 grams.

Now, 32 grams of Sulphur = 1 mole of Sulphur

So, 16 grams of Sulphur = (1/32) × 16 = 0.5 moles

Thus, 16 grams of Sulphur constitute 0.5 mole of Sulphur.

Ex.13 Which has more number of atoms, 100 grams of sodium or 100 grams of iron. Given that atomic mass of Na = 23 u, Fe = 56 u.

Sol. In case of atoms of elements, number of atoms depend upon the number of moles of the elements. Greater the number of moles, greater is the number of atoms.

$$\text{Moles of sodium} = \frac{\text{Mass in gram}}{\text{Atomic mass}} = \frac{100}{23} = 4.35$$

$$\text{Moles of iron} = \frac{\text{Mass in gram}}{\text{Atomic mass}} = \frac{100}{56} = 1.78$$

Therefore, number of particles in 100 gram sodium are more than 100 gram iron.

If we want to calculate the exact number of atoms, then multiply the number of moles by 6.022×10^{23}.

Number of sodium atoms $= 4.35 \times 6.022 \times 10^{23}$

$= 26.196 \times 10^{23}$

Number of iron atoms $= 1.78 \times 6.022 \times 10^{23}$

$= 10.72 \times 10^{23}$

Therefore, number of atoms of sodium are more than those in iron.

Ex.14 Calculate the number of moles, molecules, number of H-atoms, number of S-atoms and number of O-atoms in 49 gram H_2SO_4.

Sol. In such questions, number of moles are calculated first. Moles on multiplication with 6.022×10^{23} give the number of molecules. Then see how many atoms of the given element are present in one molecule of the compound? Then calculate the number of atoms.

Mass of H_2SO_4 (m) = 49 gram,

Molar mass of H_2SO_4(M) = 98 gram.

$$\text{Moles of } H_2SO_4 = \frac{\text{Mass in gram (m)}}{\text{Molar mass (M)}} = \frac{49}{98} = 0.5$$

Number of molecules of H_2SO_4 = Moles $\times 6.022 \times 10^{23}$

$= 0.5 \times 6.022 \times 10^{23}$

$= 3.011 \times 10^{23}$

Now one molecules of H_2SO_4 contains: Two H-atom, One S-atom, Four O-atoms

Number of H-atoms = 2 × number of molecules of H_2SO_4

$= 2 \times 3.011 \times 10^{23}$

$= 6.022 \times 10^{23}$

Number of S-atoms = 1 × Number of molecules of H_2SO_4

$= 1 \times 3.011 \times 10^{23}$

$= 3.011 \times 10^{23}$

Number of O-atoms = 4 × Number of molecules of H_2SO_4

$= 4 \times 3.011 \times 10^{23}$

$= 12.044 \times 10^{23}$

Ex.15 What is the mass of

(a) One mole of nitrogen atoms.

(b) 4 moles of aluminium atoms.

(c) 10 moles of sodium sulphite (Na_2SO_3).

(Given that N = 14, Al = 27, Na = 23, S = 32, O = 16)

Sol. (a) Mass (in gram) of N-atoms = Moles × At. mass

$= 1 \times 14 = 14$ g

(b) Mass (in gram) of Al-atoms = Moles × At.mass

$= 4 \times 27 = 108$ g

(c) Molar mass of $Na_2SO_3 = (2 \times 23) + (1 \times 32) + (3 \times 16)$

$= 126$ g

Mass of Na_2SO_3 = Moles × Molar mass

$= 10 \times 126 = 1260$ g

Ex.16 An element contains 9.033×10^{24} atoms. How many moles does it contain?

Sol. $$\text{Number of moles} = \frac{\text{Number of particles}}{6.022 \times 10^{23}}$$

$$= \frac{9.033 \times 10^{24}}{6.022 \times 10^{23}} = 15 \text{ mole}$$

Ex.17 Calculate the mass of the following:

(i) 1 atom of sodium

(ii) 10 molecules of argon

(iii) 1 molecule of CO_2

(iv) 1 molecule of H_2SO_4

Sol. (i) The Gram-atomic mass of sodium = 23 g = 6.022×10^{23} sodium atoms.

$\therefore$ Mass of one atom of sodium = $\frac{23}{6.022 \times 10^{23}} = 3.819 \times 10^{-23}$ g

(ii) Argon is monoatomic. Therefore, its one molecule contains only one atom.

Hence,

Molar mass of argon = 40 g mol^{-1} = 6.022×10^{23} molecules of argon.

$\because$ Mass of 6.022×10^{23} molecules of argon = 40 g.

$\therefore$ Mass of 10 molecules of argon = $\frac{10 \times 40}{6.022 \times 10^{23}} = 6.642 \times 10^{-22}$ g

(iii) Molar mass of CO_2 = 44 g = 6.022×10^{23} molecules CO_2.

$\therefore$ Mass of one molecule of CO_2 = $\frac{44}{6.022 \times 10^{23}} = 7.307 \times 10^{-23}$ g.

(iv) Molar mass of H_2SO_4 = 98 g = 6.022×10^{23} molecules of H_2SO_4.

$\therefore$ mass of 1 molecule of H_2SO_4 = $\frac{98}{6.022 \times 10^{23}} = 16.27 \times 10^{-23}$ g

Ex.18 Calculate the number of mole and number of molecules in 4.4 g of CO_2.

Sol. The molecular mass of CO_2 = 12 + (2 × 16) = 44u. Hence, the molar mass of CO_2 = 44 g

$\because$ Number of mole = $\frac{\text{Mass in grams}}{\text{molar mass}}$

$\therefore$ Number of mole contained in 4.4 g of CO_2 = $\frac{4.4}{44} = 0.10$

$\because$ One mole of CO_2 has 6.022×10^{23} molecules.

$\therefore$ Number of molecules in 0.10 mole of CO_2 = $0.10 \times 6.022 \times 10^{23} = 6.022 \times 10^{22}$ g

- *Number of moles* = $\frac{\text{Given volume}}{\text{molar volume}}$

  (Molar volume = 22.4L,)

Ex.19 Calculate the mass of 60 mL of chlorine at S.T.P. (Atomic mass of Cl = 35.5 u)

Sol. Molecular mass of chlorine (Cl_2) = 35.5 × 2 = 71 u

Molar mass of Cl_2 = 71 g mol^{-1}

1 mole of chlorine = 71 g = 22.4 L at S.T.P.

Therefore,

$\because$ Mass of 22400 mL Cl_2 at S.T.P. = 71 g

$\therefore$ Mass of 60 mL of Cl_2 at S.T.P. = $\frac{71}{22400} \times 60 = 0.19$ g

Ex.20 What volume will be occupied by 4.4 g of CO_2 gas at S.T.P.? (Given: C = 12, O = 16)

Sol. Molecular mass of $CO_2 = 12 + (16 \times 2) = 44$ u.

1 mole of CO_2 = 44 g mol^{-1} = 22.4 L at S.T.P.

$\because$ 44 g of CO_2 at S.T.P. occupy a volume = 22.4 L

$\therefore$ 4.4 g of CO_2 at S.T.P. will occupy a volume = $\frac{22.4}{44} \times 4.4 = 2.24$ L

The various relationships involving mole are shown in figure.

Ex.21 Calculate the number of molecules present in

(i) 100g of methane (CH_4), and

(ii) 10 L of hydrogen at S.T.P.

(Given: Atomic mass of C = 12, H = 1)

Sol. (i) Molecular mass of $CH_4 = 12 + (1 \times 4) = 16$ u

1 mole of CH_4 = 16 g $CH_4 = 6.022 \times 10^{23}$ molecules of CH_4

Therefore,

$\because$ Number of molecules in 16 g of $CH_4 = 6.022 \times 10^{23}$

$\therefore$ Number of molecules in 100 g of $CH_4 = \frac{6.022 \times 10^{23}}{16} \times 100 = 3.76 \times 10^{24}$

(ii) $\because$ Number of molecules present in 10 L of H_2 at S.T.P. = $\frac{6.022 \times 10^{23}}{22.4} \times 10 = 2.69 \times 10^{23}$

Percentage Composition

The percentage composition of elements in a compound is calculated from the molecular formula of the compound. The molecular mass of the compound is calculated from the atomic masses of the various elements present in the compound. The percentage by mass of each element is then computed with the help of the following relations.

Percentage mass of the element in the compound = $\frac{\text{Total mass of the element}}{\text{Molecular mass}} \times 100$

Ex.22 What is the percentage of calcium in calcium carbonate ($CaCO_3$)?

Sol. Molecular mass of $CaCO_3 = 40 + 12 + 3 \times 16 = 100$ amu.

Mass of calcium in 1 mol of $CaCO_3$ = 40g.

$\therefore$ Percentage of calcium = $\frac{40 \times 100}{100} = 40$ %

Determination of Empirical & Molecular Formula

The chemical formula of the compound that can be calculated from the element analysis data (percentage by mass of each element in the compound) is called the empirical formula.

Empirical formula represents the simplest whole number ratio of the atoms of different constituent elements present in one molecule of the compound.

◈ Steps taken to arrive at the empirical formula

(i) Determine the percentage of each element: Percentage of each element is determined quantitatively.

(ii) Divide the percentage of each element by its atomic mass to obtain the relative numbers of atoms (or atomic ratio).

$$\text{Atomic ratio} = \frac{\text{Percentage of an element}}{\text{Atomic mass of the same element}}$$

(iii) Divide the atomic ratio by smallest quotient to get the simplest ratio of various elements.

(iv) Convert the simplest ratio to the whole number ratio either by,

(a) Rounding them off to whole number, if the value is quite close to whole number e.g.: 1.98, 2.99, 3.95 are rounded off as 2, 3 and 4 respectively.

(b) Multiply the figures by suitable integer (2, 3 or 4 etc.)

(v) Write the empirical formula of the compound by writing the symbols of the various elements side by side. Now insert the whole number ratio of each element as the subscripts to the lower right hand corner of each symbol.

Ex.23 An inorganic salt gave the following percentage composition Na = 29.11, S = 40.51 and O = 30.38. Calculate the empirical formula of the salt.

Sol. Calculation of empirical formula:

Element	Symbol	Percentage of element	At. mass of elements	Moles of the element = Percentage At. mass (Relative no. of moles)	Simplest molal ratio	Simplest whole no. molar ratio
Sodium	Na	29.11	23	$\frac{29.11}{23} = 1.266$	$\frac{1.266}{1.266} = 1$	2
Sulphur	S	40.51	32	$\frac{40.51}{32} = 1.266$	$\frac{1.266}{1.266} = 1$	2
Oxygen	O	30.38	16	$\frac{30.38}{16} = 1.897$	$\frac{1.89}{1.266} = 1$	3

Thus, the Empirical formula is $Na_2S_2O_3$.

◈ Molecular Formula

It is that formula of the compound which gives the actual number of atoms of various elements in a molecules of that compound.

For example, molecular formula of glucose is $C_6H_{12}O_6$. Molecular formula of a compound is related to empirical formula as being a simple whole number multiple of empirical formula. The molecular and empirical formula of any chemical compound are related as a n × Empirical formula = Molecular formula, where 'n' is positive integer.

$$n = \frac{\text{Molecular mass of compound}}{\text{Empirical formula mass}}$$

e.g.: Let us consider hydrogen peroxide whose empirical formula is HO and its molecular formula mass is 34.

$$N = \frac{\text{Molecular formula mass of hydrogen peroxide}}{\text{Empirical formula mass of hydrogen peroxide}} = \frac{34}{1+16} = \frac{34}{17} = 2$$

Thus, the molecular formula of hydrogen peroxide = $(\text{empirical formula})_n = (HO)_2 = H_2O_2$.

When n = 1.

Molecular formula = Empirical formula

The molecular mass of a volatile compound can be determined by Victor Meyer's method (based on the principle that 22.4 L of vapours of a substance at STP have mass equal to gram molecular mass) or by employing the relation.

Molecular mass = 2 × Vapour density.

Limiting Reagent

In many situations, an excess of one or more substance is available for chemical reaction. Some of these excess substances will therefore be left over when the reaction is complete; the reaction stops immediately as soon as one of the reactant is totally consumed.

The substance that is totally consumed in a reaction is called limiting reagent because it determines or limits the amount of product. The other reactant present in excess are called as excess reagents.

Let us consider a chemical reaction which is initiated by passing a spark through a reaction vessel containing 10 mole of H_2 and 8 mole of O_2.

$$2\,H_2(g) + O_2(g) \rightarrow 2H_2O$$

The reaction stops only after consumption of 5 moles of O_2 as no further amount of H_2 is left to react with unreacted O_2. Thus H_2 is a limiting reagent in this reaction.

Ex.24 8 litre of H_2 and 6 litre of Cl_2 are allowed to react to maximum possible extent. Find out the final volume of reaction mixture. Suppose P and T remains constant throughout the course of reaction.

Sol.

	H_2	+ Cl_2	$\rightarrow$ 2HCl
Volume before reaction	8 litre	6 litre	
Volume after reaction	2	0	12 litre

here, Cl_2 is a limiting reagent

hence

Volume after reaction = Volume of H_2 left + volume of HCl formed = 2 + 12 = 14 litre.

Ex.25 10 g N_2 is reacting with 20 g of H_2 to form ammonia then find limiting reagent in this reaction is also find the maximum amount of NH_3 formed.

Sol. $N_2 + 3H_2 \rightarrow 2NH_3$

no. of moles of N_2 $(n_{N_2}) = \frac{10}{28} = 0.357$ mole

no. of moles of H_2 $(n_{H_2}) = \frac{20}{2} = 10$ mole

$\because$ 1 mole if N_2 is reacting with 3 mole of H_2.

$\therefore$ 0.357 mole of N_2 would react = 3 × 0.357

= 1.078 mole of H_2

Hence N_2 is L.R.

Formation of ammonia will depend on L.R.

$\because$ 1 mole of N_2 forming 2 mole of NH_3

$\therefore$ 0.357 mole of N_2 would form = 2×0.357

= .714 mole of NH_3

Wt. of $NH_3 = 0.714 \times 17 = 12.138$ g

EXERCISE-1

➢ Very Short Answer Type Questions

Q.1 Name of the building block of all matter.

Q.2 What is the molecular mass of H_2SO_4?

Q.3 Which of the following is tetraatomic molecule CH_3OH, CH_4, H_2O_2 ?

Q.4 "If 100 grams of calcium carbonate are decomposed completely, then 56 grams of calcium oxide and 44 grams of carbon dioxide are obtained" Which law of chemical combination is illustrated by this statement?

Q.5 An element X of valency 3 combines with another element Y of valency 2. What will be the formula of the compound formed?

➢ Short Answer Type Questions – Type I

Q.6 Name two elements whose symbols are derived from Latin names. Give their symbols.

Q.7 Calculate the number of moles of phosphorus molecule in 12.044×10^{25} atoms of phosphorus.

Q.8 An element B shows valencies of 4 and 6. Write the formulae of its two oxides.

Q.9 What do the symbols, H_2, S and O_4 mean in the formula H_2SO_4?

Q.10 Give two examples of trivalent metal ions.

➢ Short Answer Type Questions – Type II

Q.11 What is the ratio by mass of nitrogen and hydrogen in ammonia?

Q.12 Define 'formula unit mass' of an ionic compound. What is the formula unit of
(i) sodium chloride
(ii) magnesium chloride

Q.13 The mass of one atom of an element X is 2.0×10^{-23} g.
(i) Calculate the atomic mass of element X.
(ii) What could element X be?

Q.14 The mass of one molecule of a substance is 4.65×10^{-23} g.
(i) What is its molecular mass?
(ii) What could this substance be?

Q.15 What weight of oxygen gas will contain the same number of molecules as 56 g of nitrogen gas? (O = 16 u ; N = 14 u)

Q.16 Define mole with example. What is meant by 1 mole of carbon atoms?

Q.17 (i) What is an ion? How is an ion formed?
(ii) What is the difference between a cation and an anion? Explain with examples.

Q.18 Explain the difference between 2N and N_2.

Q.19 Which of the following has more number of atoms and by how much?
(i) A mole of ammonia (NH_3)
(ii) A mole of methane (CH_4)

Q.20 Write the chemical formulae of the following
(i) Aluminium hydroxide
(ii) Ammonium suphate
(iii) Potassium carbonate

➢ Long Answer Type Questions

Q.21 Give the main postulates of Dalton's atomic theory. Which postulate of Dalton's atomic theory explain the law of conservation of mass.

Q.22 Calculate the following:
(i) Number of S atoms in 3.2 g of S_8.
(ii) Number of molecules of CH_4 in 80.0 g of it
(iii) The mass of 1 molecule of NH_3.
(iv) The mass of 0.25 moles of calcium
(v) Number of bromide ion in 0.2 mole of $MgBr_2$.

Q.23 Calculate the mass of
(a) 0.2 mol of O_2
(b) One atom of aluminium
(c) 3.0 mole of chloride ion
(d) 2.5 mol of NaCl

Q.24 Write the formula and names of compounds formed by
(a) Na^+ and HCO_3^- (b) K+ and CO_3^{2-}
(c) Cu^{2+} and SO_4^{2-} (d) Cu^{2+} and O^{2-}
(e) Na^+ and SO_4^{2-} (f) NH_4^+ and CO_3^{2-}

Q.25 Calculate the number of moles of phosphorus (P) atoms in 155 g of phosphorus. If phosphorus is considered to contain P_4 molecules, then how many moles of P_4 molecules are there?

EXERCISE-2

Q.1 Atomic radius of atoms is measured in nanometers (nm). One nanometer is equal to-
(A) 10^{-10} metre (B) 10^{-9} metre
(C) 10^{-10} cm (D) 10^{-9} cm

Q.2 Symbol of which element has been derived from its Latin name?
(A) Sodium (B) Iron
(C) Copper (D) All

Q.3 Symbol of which element has been derived from its English name?
(A) Hydrogen (B) Carbon
(C) Aluminium (D) All

Q.4 One unified mass (u) is equal to -
(A) 1.66×10^{-24} g (B) 1.66×10^{-23} g
(C) 1.66×10^{-24} kg (D) 1.66×10^{-23} kg

Q.5 Polyatomic ion has -
(A) More than one unit negative charge
(B) More than one unit positive charge
(C) Either more than one unit negative charge or positive charge
(D) Two or more atoms having charge

Q.6 Which of the following are monovalent ions?
(A) Chloride and sulphate
(B) Nitrite and nitrate
(C) Carbonate and sulphide
(D) Oxide and bromide

Q.7 Which of the following are divalent ions?
(A) Carbonate and sulphide
(B) Sulphide and oxide
(C) Sulphate and cupric
(D) All

Q.8 Which of the following are trivalent ions?
(A) Cupric and ferric
(B) Mercuric and phosphate
(C) Ferric and phosphate
(D) Aluminium and sulphate

Q.9 Correct name is -
(A) CO is named as carbon monoxide
(B) PCl_3 is named as phosphorus chloride
(C) N_2O_5 is named as dinitrogen oxide
(D) HCl is named as hydrogen monochloride

Q.10 Incorrect name is -
(A) PCl_5 is named as phosphorus pentachloride
(B) N_2O_4 is named as dinitrogen tetraoxide
(C) H_2S is named as dihydrogen sulphide
(D) HCl is named as hydrogen chloride

Q.11 Correct formula is -
(A) $Al_2(SO_4)_3$ and $CaCO_3$
(B) Ca_3PO_4 and NaCl
(C) $Ca(HCO_3)_2$ and KSO_4
(D) $CaCl_2$ and $NaCO_3$

Q.12 Incorrect formula is -
(A) $Al_2(CO_3)_3$ and $CaCl_2$
(B) Na_2NO_3 and FeCl
(C) $FeSO_4$ and CuCl
(D) $HgCl_2$ and $FeCl_3$

Q.13 Incorrect statement is -
(A) One mole = 6.022×10^{23} particles
(B) Gram atomic mass = mass of 6.022×10^{23} particles in gram
(C) One mole of H_2SO_4 has 6.022×10^{23} atoms
(D) Mass of 6.022×10^{23} molecules of ammonia is 17 gram

Q.14 Oxide and sulphide ions are respectively -
(A) Both are divalent
(B) Univalent and divalent
(C) Divalent and univalent
(D) Both are trivalent

Q.15 True statements is -
(A) Cation has more electrons than its neutral atom.
(B) Anion has less electrons than its neutral atom
(C) Cation and anion of the same element contain equal number of electrons
(D) Cation has less electrons while anion has more electrons than their neutral atoms

Q.16 Which of the following would weigh the highest?
(A) 0.2 mole of sucrose ($C_{12}H_{22}O_{11}$)
(B) 2 moles of CO_2
(C) 2 moles of $CaCO_3$
(D) 10 moles of H_2O

Q.17 Which of the following has maximum number of atoms?
(A) 18 g of H_2O (B) 18 g of O_2
(C) 18 g of CO_2 (D) 18 g of CH_4

Q.18 Which of the following contains maximum number of molecules?
(A) 1 g CO_2 (B) 1 g N_2
(C) 1 g H_2 (D) 1 g CH_4

Q.19 Mass of one atom of oxygen is –
(A) $\frac{16}{6.023\times10^{23}}$ g (B) $\frac{32}{6.023\times10^{23}}$ g
(C) $\frac{1}{6.023\times10^{23}}$ g (D) 8 u

Q.20 A change in the physical state can be brought about -
(A) only when energy is given to the system
(B) only when energy is taken out from the system
(C) when energy is either given to or taken out from the system
(D) without any energy change

Q.21 The law of chemical combination that is applied when two elements combine to forms two or more compounds -
(A) Law of conservation of mass
(B) Law of definite proportion
(C) Law of multiple proportion
(D) all the three laws (a, b and c)

Q.22 The law of conservation of mass is seen in -
(A) an unbalanced chemical equation
(B) a balanced chemical equation
(C) both (A) and (B)
(D) none of these

Q.23 The law of multiple proportion can be applied to which compounds out of NO_2, CO_2, SO_2, SO_3, N_2O
(i) CO_2, SO_2 (ii) CO_2, SO_3
(iii) SO_3, H_2O (iv) SO_2, SO_3
(v) NO_2, N_2O
(A) i and ii (B) ii and iii
(C) iii and iv (D) iv and v

Q.24 Matter can neither be created nor destroyed in a chemical reaction. The statement is in accordance with the -
(A) Law of Constant Proportions
(B) Law of Conservation of Mass
(C) Law of Multiple Proportions
(D) none of these

Q.25 If the ratio of weights of oxygen in two oxides of an element, that combines with a fixed weight of the element is 1: 2, then the ratio of weights of the element that combines with a fixed weight of oxygen is -
(A) 1: 2 (B) 2: 1 (C) 4: 1 (D) 1: 4

Q.26 The molecular formula of ethanoic acid is $C_2H_4O_2$. Its empirical formula is -
(A) $C_4H_8O_2$ (B) CH_2O
(C) CHO (D) CHO_2

Q.27 Chemical analysis of a carbon compound gave the following percentage composition by weight of the elements present in it. Carbon = 10.06 %, hydrogen = 0.84 %, chlorine = 89.10 %. Calculate the empirical formula of the compound -
(A) $C_2H_2Cl_2$ (B) $CHCl_2$
(C) $CHCl_3$ (D) $C_4H_4Cl_4$

Q.28 Percentage of carbon is highest in -
(A) CH_4 (B) C_3H_4 (C) C_6H_6 (D) C_3H_8

Q.29 1 mole of SO_2 and 1 mole of H_2S react completely to form
$SO_2 + 2H_2S \rightarrow 2H_2O + 3S$ (S = 32, O = 16)
(A) 96 g S (B) 48 g S
(C) 24 g S (D) 64 g S

Q.30 $2H_2 + O_2 \rightarrow 2H_2O$
2g H_2 and 1g O_2 reacts to form H_2O:
(A) 3.0 g (B) 1.125 g
(C) 4.5 g (D) 2.50 g

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

Q.1 In a chemical reaction one molecule of hydrogen sulphide gas reacts with two molecules of nitric acid. The molecules of nitrogen dioxide and water and atoms of sulphur formed has a ratio of -
(A) 1: 2: 2 (B) 2: 1: 2
(C) 2: 2: 1 (D) 1: 1: 1

Q.2 Molecular formula of a compound of metal M (equivalent weight 8) is M_2CO_3. Atomic weight of the metal will be –
(A) 24 (B) 16 (C) 8 (D) 4

Q.3 The quantity of potassium chlorate (molecular weight 122.5) required in grams to produce 33.6 litre of oxygen at normal temperature and pressure will be -
(A) 122.5 (B) 132.6
(C) 245.0 (D) 254.0

Q.4 The group of symbols of elements based on Latin names out of the following is -
(A) Ca, Ag, Cu
(B) Ag, Au, K
(C) Pb, Cl, Cd
(D) Cs, Fe, Na

Q.5 The molecular formula of the compound formed by the combination of elements $_{13}A^{27}$ and $_{8}B^{16}$ is:
(A) AB (B) A_2B (C) AB_2 (D) A_2B_3

Q.6 1.204×10^{24} molecules are present in a sample of methane. What will be its weight (in grams)?
(A) 32 (B) 24 (C) 16 (D) 12

Q.7 $Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$
According to the above reaction, how many grams of Na_2CO_3 will be needed to obtain 11.2 litre CO_2?
(A) 51 (B) 53 (C) 83 (D) 106

Q.8 The empirical formula of a compound is CH_2. If its vapour density be 21, the molecular formula of this compound will be -
(A) C_2H_4 (B) C_3H_6
(C) C_4H_8 (D) C_5H_{10}

Q.9 A gas X is five times heavier than hydrogen gas while the other gas Y is two times heavier than X. The molecular weight of Y will be –
(A) 2.5 (B) 5
(C) 20 (D) 10

Q.10 Which one of the following is cation ?
(A) Na^+
(B) Cl^-
(C) H_2
(D) None of the above

Q.11 One Litre of a gas at STP has mass equal to 1.25 g. The gas is -
(A) H_2 (B) O_2
(C) N_2 (D) CO_2

Q.12 2 litre of hydrogen and 1.2 litre of chlorine are mixed and exploded.The composition by volume of the resultant mixture will be -
(A) 0.8 litre of hydrogen and 2.4 litre of chlorine
(B) 0.8 litre of hydrogen and 2.4 litre of hydrogen chloride
(C) 2.4 litre of hydrogen chloride
(D) 0.8 litre of chlorine and 24 litre of hydrogen chloride

Q.13 When 2 moles of N_2 gas and 9 moles of H_2 gas are mixed and reaction is completed to from NH_3 gas then reaction mixture will contain:
(A) 11 moles of NH_3
(B) 4 moles of NH_3 + 3 moles of H_2
(C) 6 moles of NH_3 + 3 moles of H_2
(D) 2 moles of NH_3 + 1 mole of N_2

Q.14 What mass of hydrogen and oxygen will be produced on complete electrolysis of 18g of water?
(A) 2g hydrogen and 32g oxygen
(B) 2g hydrogen and 16g oxygen
(C) 4g hydrogen and 32g oxygen
(D) 4g hydrogen and 14g oxygen

Q.15 Number of molecules in 14 g of carbon monoxide is –
(A) 12.044×10^{23}
(B) 6.022×10^{23}
(C) 3.011×10^{23}
(D) 1.5050×10^{23}

Q.16 Which of the following molecule has an atomicity of four:
(A) H_2O (B) NH_3 (C) CH_4 (D) CO_2

Q.17 One mole of CO_2 means:
(A) 4.4 gm CO_2
(B) 2.24 litres gas at STP
(C) 6.022×10^{23} molecules of CO_2
(D) 22 gm CO_2

Q.18 Total number of atoms in 4 gms of oxygen molecule is -
(A) 6.022×10^{23} (B) 7.52×10^{22}
(C) 1.5055×10^{23} (D) 0.0752×10^{23}

Q.19 Number of molecules present in 14 gm of N_2 molecule is:
(A) 6.022×10^{23} (B) 3.011×10^{23}
(C) 1.51×10^{23} (D) 6.022×10^{22}

Q.20 Formula of aluminimum carbonate is:
(A) $Al_2(CO_3)_3$ (B) Al_2CO_3
(C) Al_2HCO_3 (D) $AlCO_3$

Q.21 Number of molecules present in 32g of O_2 is
(A) 6.022×10^{23} (B) 3.011×10^{23}
(C) 1.51×10^{23} (D) 6.022×10^{22}

Q.22 The formula of chloride of an element X is XCl_3. The formula of its oxide will be -
(A) XO_2 (B) XO_3
(C) X_2O_3 (D) X_3O_2

Q.23 Write chemical formula of Magnesium Chloride
(A) $MgCl_2$ (B) $CaCl_2$
(C) $Cu(NO_3)_2$ (D) $CaCO_3$

ANSWER KEY

EXERCISE - 1

2. 98 g

7. 50 moles

11. 14 : 3

13. (i) 12.044 g

14. (i) 28 g

15. 64 g

19. (i) 24.088×10^{23} (ii) 30.11×10^{23}
CH_4 has 6.022×10^{23} more atoms than NH_3

22. (i) 0.075×10^{23} (ii) 30.11×10^{23}
(iii) 2.82×10^{-23} g (iv) 10 g
(v) 2.40×10^{23}

23. (i) 6.4 g (ii) 4.48×10^{-23} g
(iii) 106.5 g (iv) 146.25 g

25. Number of moles of phosphorus atom = 5
Number of moles of phosphorus molecule = 1.25

EXERCISE - 2

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	D	D	A	D	B	D	C	A	C	A	B	C	A	D
Ques.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	D	C	A	C	C	B	D	B	B	B	C	C	B	B

EXERCISE - 3

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	C	A	B	D	A	B	B	C	A	C	B	B	B	C
Ques.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**							
Ans.	B	C	C	B	A	A	C	A							

SOLUTIONS

EXERCISE-1

➢ Very Short Answer Type Questions

Sol.1 Atoms.

Sol.2 H_2SO_4
Molar mass = (2 × 1) + (1 × 32) + (4 × 16)
= 98 g

Sol.3 H_2O_2
Atomicity is 4.

Sol.4 $CaCO_3 \longrightarrow CaO + CO_2$
100g 56g 44g
Law of conservation of mass.

Sol.5 Low of mass conservation is obeyed
Element X has valency 3
Element Y has valency 2
Fromula X_2Y_3 (Criss cross method)

➢ Short Answer Type Questions – Type I

Sol.6 Latin names :
Sodium – Natrium - Na
Potassium – Kalium - K

Sol.7 1 mole of phosphorus molecule,
$P_4 = N_A$ molecules
$= 4N_A$ atoms $= 4 \times 6.022 \times 10^{23}$
Moles of phosphorus molecule in 12.044×10^{25} atoms

$$= \frac{12.044 \times 10^{25}}{4 \times 6.022 \times 10^{23}}$$

$$= \frac{12.044 \times 10^{25}}{24.088 \times 10^{23}} = \frac{1}{2} \times 10^2 = 50 \text{ moles}$$

Sol.8 Valency 4 = BO_2
Valency 6 = BO_3

Sol.9 Two hydrogen atoms (H_2), 1 sulphur atom and 4 oxygen atoms. Combine to form 1 molecule of H_2SO_4.

Sol.10 Al^{+3}, Fe^{+3}, Mn^{3+}, CO^{3+}, Cr^{3+}.

➢ Short Answer Type Questions – Type II

Sol.11 Ammonia, NH_3
Mass of Nitrogen = 14g
Mass of Hydrogen = 3g.
Mass of nitrogen and hydrogen in ammonia is in the ratio of 14 : 3.

Sol.12 Formula unit mass : Formula mass means the sum of the masses of all the species in the formula.
(i) NaCl 23 + 35+5 = 58.5g
(ii) $MgCl_2$ 24 + 2(35.5) = 24 + 71 = 95g

Sol.13 Mass of one atoms X = 2×10^{-23} g
Mass of 6.023×10^{23} atom = ?

$$= \frac{2 \times 10^{-23} \text{ g}}{1 \text{ atom}} \times 6.023 \times 10^{23} \text{ atom}$$

= 12.046 g
(i) Atomic mass of X = 12 g
(ii) Element is carbon.

Sol.14 Mass of 1 molecule = 4.65×10^{-23} g
Mass of 6.023×10^{23} molecule

$$= \frac{4.65 \times 10^{-23} \text{ g}}{1 \text{ molecule}} \times 6.023 \times 10^{23} = 28 \text{ g}$$

(i) Molar mass of molecule = 28 g
(ii) Nitrogen

Sol.15 Molecules in 56g of $N_2 = \frac{6.023 \times 10^{23}}{28} \times 56$
$= 12.046 \times 10^{23}$
N_A molecule in 32 g O_2
Mass of 12.046×10^{23} molecule in 32 g O_2

$$= \frac{12.046 \times 10^{23}}{6.023 \times 10^{23}} \times 32 = 64 \text{ g}$$

Sol.16 Mole : Amount of substance present in 6.023×10^{23} particles of atom or molecule or ion.
1 mole of carbon atom means it contains 6.023×10^{23} atoms of carbon.

Sol.17 (i) An ion is a positively or negatively charged atom (or group of atoms). An ion is formed by the loss or gain of electrons by an atom.

(ii)

Cation	Anion
Positively charged ion where no. of protons are greater than electrons. Eg. Na^+, Al^3	Negatively charged ion where no. of electrons are greater than no. of protons. Eg. Cl^-, SO_4^{2-}

Sol.18 2N is for 2 nitrogen atom and it contains $2 \times N_A$ no. of atoms.
N_2 is molecule that contains N_A of molecules of nitrogen.

Sol.19 1 mole of $NH_3 = 4N_A$ atoms
Atomicity of $NH_3 \rightarrow 4$
1 mole of CH_4 = 5 NA atoms
Atomicity of $CH_4 \rightarrow 5$
A mole of methane (CH_4) has N_A (6.022×10^{23}) more atoms then a mole of ammonia (NH_3).

Sol.20 Aluminium hydronide – $Al(OH)_3$
Ammonium sulphate – $(NH_4)_2SO_4$
Potassium carbonate – K_2CO_3.

➢ Long Answer Type Questions

Sol.21 The basic postulates of Dalton's theory are as follows:
(i) Each element is composed of extremely small particles called atoms.
(ii) All atoms of a given element are identical i.e., atoms of a particular element are alike in all aspects.
(iii) Atoms of different elements possess completely different properties and differ in all aspects.
(iv) Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions.
(v) Atoms of element take part in chemical reaction to form molecules.
(vi) In a given compound, the relative number and kind of atoms are constant.
(vii) Atom of same type of elements combine in different ratio to form more than one type of compounds.
Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions. This postulate of Dalton's atomic theory explain the law of conservation of mass.

Sol.22 (i) Number of S atoms in 3.2 g of S_8
Molar mass = $8 \times 32 = 256$ g
256 g contain $8 \times N_A$ atoms

$$3.2 \text{ g contain} = \frac{8 \times 6.023 \times 10^{23}}{256\text{g}} \times 3.2 \text{ g}$$

$$= 6.023 \times 10^{22}$$

(ii) Number of molecules of CH_4 in 80 g of it
16 g contain N_A molecule

$$\therefore 80 \text{ g contain} = \frac{6.023 \times 10^{23}}{16} \times 80$$

$$= 30.11 \times 10^{23}$$

(iii) The mass of 1 molecule of NH_3
17g contains N_A molecule

$$\therefore 1 \text{ molecule is} = \frac{17}{6.023 \times 10^{23}} \times 1$$

$$= 2.8 \times 10^{-23}$$

(iv) 1 mole of Ca = 40g
$\therefore$ 0.25 mole of Ca = $40 \times 0.25 = 10$ g
(v) $MgBr_2 \rightleftharpoons Mg^{2+} + 2Br^-$
1 mole of MgBr contain 2 mole of bromide ions
1 mole of Bromide ions = 6.022×10^{23} bromide ion
0.2 mole of $MgBr_2$ will have 0.4 mole of bromide ion
Bromide ions in 0.2 mole of $MgBr_2$
$= 0.4 \times 6.022 \times 10^{23}$
$= 2.40 \times 10^{23}$ bromide ions

Sol.23 (a) 1 mole of O_2 = 32 g

$$0.2 \text{ mol of } O_2 = \frac{32\text{g}}{1\text{mole}} \times 0.2 \text{ mole} = 6.4 \text{ g}$$

(b) One atom of aluminium
N_A of Al contain = 27 g

$$1 \text{ atom contain} = \frac{27}{6.023 \times 10^{23}} = 4.5 \times 10^{-23}$$

(c) 1 mole of Cl^- = 35.5 g
3 mole of Cl^- = $3 \times 35.5 = 106.5$ g
(d) 1 mole of NaCl weighs 58.5 g
2.5 mole of NaCl = $58.5 \times 2.5 = 146.25$ g

Sol.24 (a) $NaHCO_3$ (b) K_2CO_3
(c) $CuSO_4$ (d) CuO
(e) Na_2SO_4 (f) $(NH_4)_2CO_3$

Sol.25 Molar mass of phosphorus atom, P = 31
Molar mass of phosphorus molecule,
$P_4 = 4 \times 31 = 124$
Given, mass of phosphorus = 155 g
$\therefore$ No of moles phosphorus atom,
P = 155/31 = 5 moles.
$\therefore$ No of moles phosphorus molecule,
P_4 = 155/124 = 1.25 moles.

EXERCISE-2

Sol.1 [B]
One nm = 10^{-9} m.

Sol.2 [D]
Latin name – Sodium-Natrium,
Iron-Ferrum, Copper-Cuprum.

Sol.3 [D]
English name – Carbon, Hydrogen, Aluminium.

Sol.4 [A]
One unified mass = 1.66×10^{-24} g.

Sol.5 [D]
Polyatomic ion has two or more atoms having charge.

Sol.6 [B]
NO_2^- & NO_3^- monovalent ions.

Sol.7 [D]
CO_3^{2-}, S^{2-}, O^{2-}, SO_3^{2-}, Cu^{2+} all divalent ions.

Sol.8 [C]
Fe^{3+} & PO_4^{3-} – Trivalent ions.

Sol.9 [A]
CO ⇒ Carbon monoxide.

Sol.10 [C]
H_2S ⇒ Hydrogen sulphide.

Sol.11 [A]
$Al_2(SO_4)_3$ Aluminium sulphate
$CaCO_3$ Calcium carbonate.

Sol.12 [B]
Incorrect formulae are Na_2NO_3 & FeCl.
Correct formula $NaNO_3$ & $FeCl_3$.

Sol.13 [C]
One mole of H_2SO_4 contains
$7 \times 6.022 \times 10^{23}$ atoms.

Sol.14 [A]
Oxide (O^{2-}) & S^{2-} sulphide ⇒ divalent.

Sol.15 [D]
True statements is –
Cation has less electrons
Anion has more electrons.

Sol.16 [C]
2 moles of $CaCO_3$
Molar mass = 40 + 12 + 48 = 100 g
Mass of 2 moles = $2 \times 100 = 200$ g

Sol.17 [D]
Maximum no. of atoms
18 g of CH_4
16 g contain $5 \times 6.023 \times 10^{23}$ atoms
18 g contain $= \frac{5 \times 6.023 \times 10^{23}}{16} \times 18$
$= 33.75 \times 10^{23}$ atoms

Sol.18 [C]
Maximum no. of molecules
1 g H_2 2g $H_2 = N_A$ molecule
$\therefore$ 1g $H_2 = \frac{6.022}{2} \times 10^{23}$ molecules
$= 3.011 \times 10^{23}$ molecules

Sol.19 [A]
Mass of 1 atom of oxygen
$\frac{16}{6.023 \times 10^{23}}$ g $= 2.66 \times 10^{-23}$.

Sol.20 [C]
Energy given or taken causes for change in physical state of the system.

Sol.21 [C]
According to law of multiple proportion when two or more elements combines to form more than two compounds, the various masses of one element that combine with fixed mass of other element can bear a simple ratio.

Sol.22 [B]
The law of conservation of mass is seen in balanced chemical equation, $M_P = M_R$.

Sol.23 [D]
The law of multiple proportion

SO_2 & SO_3	NO_2 & N_2O
$S + O_2 \rightarrow SO_2$	$N_2 + 2O_2 \rightarrow 2NO_2$
$2S + 3O_2 \rightarrow 2SO_3$	$N_2 + \frac{1}{2}O_2 \rightarrow N_2O$

Sol.24 [B]
The law of conservation of mass matter is neither created nor destroyed.

Sol.26 [B]
Molecular formula of ethanioc acid = $C_2H_4O_2$
Empirical formula = $C_2H_4O_2 = CH_2O$

Sol.27 [C]

Elements	%	Mole	Mole (simple ratio)	Wholes (ratio mole)
C	10.06	$\frac{10.06}{12}$	0.84	$\frac{0.84}{0.84} = 1$
H	0.84	$\frac{0.84}{1}$	0.84	$\frac{0.84}{0.84} = 1$
Cl	89.10	$\frac{89.1}{35.5}$	2.5	$\frac{2.5}{0.84} = 3$

Empirical formula of compound = $CHCl_3$.

Sol.28 [C]
Benzene, C_6H_6
% carbon = $\frac{72}{78} \times 100 = 92.3\%$

Sol.29 [B]
$SO_2 + 2H_2S \rightarrow 2H_2O + 3S$
1 mole of SO_2 gives 3 moles of sulphur
2 mole of H_2S gives 3 mole of sulphur
H_2S is limiting reagent
2 mole $H_2S \rightarrow$ 3 moles 'S'
$\therefore$ 1 mole $H_2S \rightarrow \frac{3}{2} \times 1 = 1.5$ mole
$1.5 \times 32 = 48$ g

Sol.30 [B]
$2H_2 + O_2 \rightarrow 2H_2O$
2g $H_2 \rightarrow$ 18g water
32g $O_2 \rightarrow$ 36g water
1 g $O_2 \rightarrow \frac{36}{32} = \frac{9}{8} = 1.125$ g

EXERCISE-3

Sol.1 [C]
$2HNO_3 + H_2S \Rightarrow 2NO_2 + 2H_2O + S$
2 : 2 : 1

Sol.2 [C]
Molecular formula of a compound = M_2CO_3
Equivalent weight = 8
Valency = 1
Atomic weight = 8

Sol.3 [A]
$2KCl_3 \longrightarrow 2KCl + 3O_2$
$122.5 \times 2 \longrightarrow 3 \times 22.4$ litre
? $\longrightarrow$ 33.6 litre
Mass of $KClO_3 = \frac{122.5 \times 2}{3 \times 22.4} \times 33.6$
$= \frac{245.0 \times 33.6}{67.2} = \frac{8232}{67.2}$
$= 122.5$

Sol.4 [B]
Latin name
Ag – Argentum
Au – Aurram
K – Kalium

Sol.5 [D]
$_{13}A^{27}$ and $_8B^{16}$
A = Aluminium B = Oxygen
Valency = 3 Valency = 2
$\therefore$ MF = $Al_2O_3 = A_2B_3$

Sol.6 [A]
16 g in 6.023×10^{23} molecules
1.204×10^{24} molecule = $\frac{1.204 \times 10^{24}}{6.023 \times 10^{23}} \times 16$
= 32 g

Sol.7 [B]
$Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$
106 g of $Na_2CO_3 \longrightarrow$ 44 g CO_2 = 22.4 litre
11.2 litre $CO_2 \longrightarrow$ 53 g.

Sol.8 [B]
Molecular mass = 2 × V.D.
= 2 × 21 = 42
EF = CH_2
EF mass = 14
n = MF mass/ EF mass
= 42/14 = 3
MF = 3(EF) = C_3H_6.

Sol.9 [C]
Weight of X = 5 × 2 = 10
Weight of Y = 2 × 10 = 20.

Sol.11 [C]
1 litre $\longrightarrow$ 1.25 g
22.4 litre $\longrightarrow \frac{1.25}{1} \times 22.4 = 28$ molar mass
Nitrogen (N_2) has molar mass = 28.

Sol.12 [B]

$H_2 + Cl_2 \longrightarrow 2HCl$

1.2 litre $Cl_2 \longrightarrow$ 2.4 litre HCl

1.2 litre $H_2 \longrightarrow$ 2.4 litre HCl

Remaining mixture = 0.8 lit. H_2 & 2.4 lit. HCl.

Sol.13 [B]

When 2 moles of N_2 gas and 9 moles of H_2 gas are mixed and

$N_2 + 3H_2 \rightarrow NH_3$

1 3 2

2 9 $\rightarrow$? N_2 liming reagent

Remains : 3 mole H_2 + 4 mole NH_3.

Sol.14 [B]

$H_2O \rightarrow H_2 + \frac{1}{2} O_2$

2g H_2 + 16g O_2

Sol.15 [C]

Molar mass of CO = 28

28 g contains 6.023×10^{23} molecules

$\therefore$ 14 g contains 3.011×10^{23} molecules

Sol.16 [B]

NH_3 atoms = 4.

Sol.17 [C]

One mole $\rightarrow$ Avogadro no. of molecules

6.023×10^{23} molecule.

Sol.18 [C]

Molar mass of O_2 = 32

1 mole of $O_2 = 6.022 \times 10^{23} \times 2$ atoms

Given 4 g oxygen means $\frac{1}{8}$ mole

Number of atoms $= 6.022 \times 10^{23} \times 2 \times \frac{1}{8}$

$= 1.5055 \times 10^{23}$

Sol.19 [B]

Molar mass of N_2 = 28

14 g contains 3.011×10^{23} molecule.

Sol.20 [A]

Aluminium carbonate, $Al_2(CO_3)_3$.

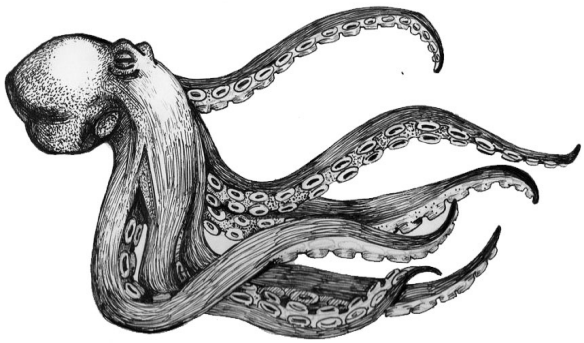
BIOLOGY

Chapter

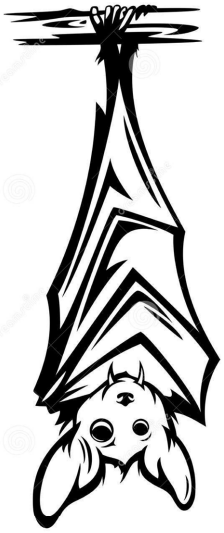
4 PATTERNS IN LIFE : DIVERSITY AND CLASSIFICATION

Chapter Outline

- Taxonomy
- Hierarchy
- Nomenclature
- Classification
- Kingdom Monera
- Kingdom Protista
- Kingdom Plantae
- Kingdom Animalia
- Viruses
- Discovery of Viruses
- Structural Organization of Viruses



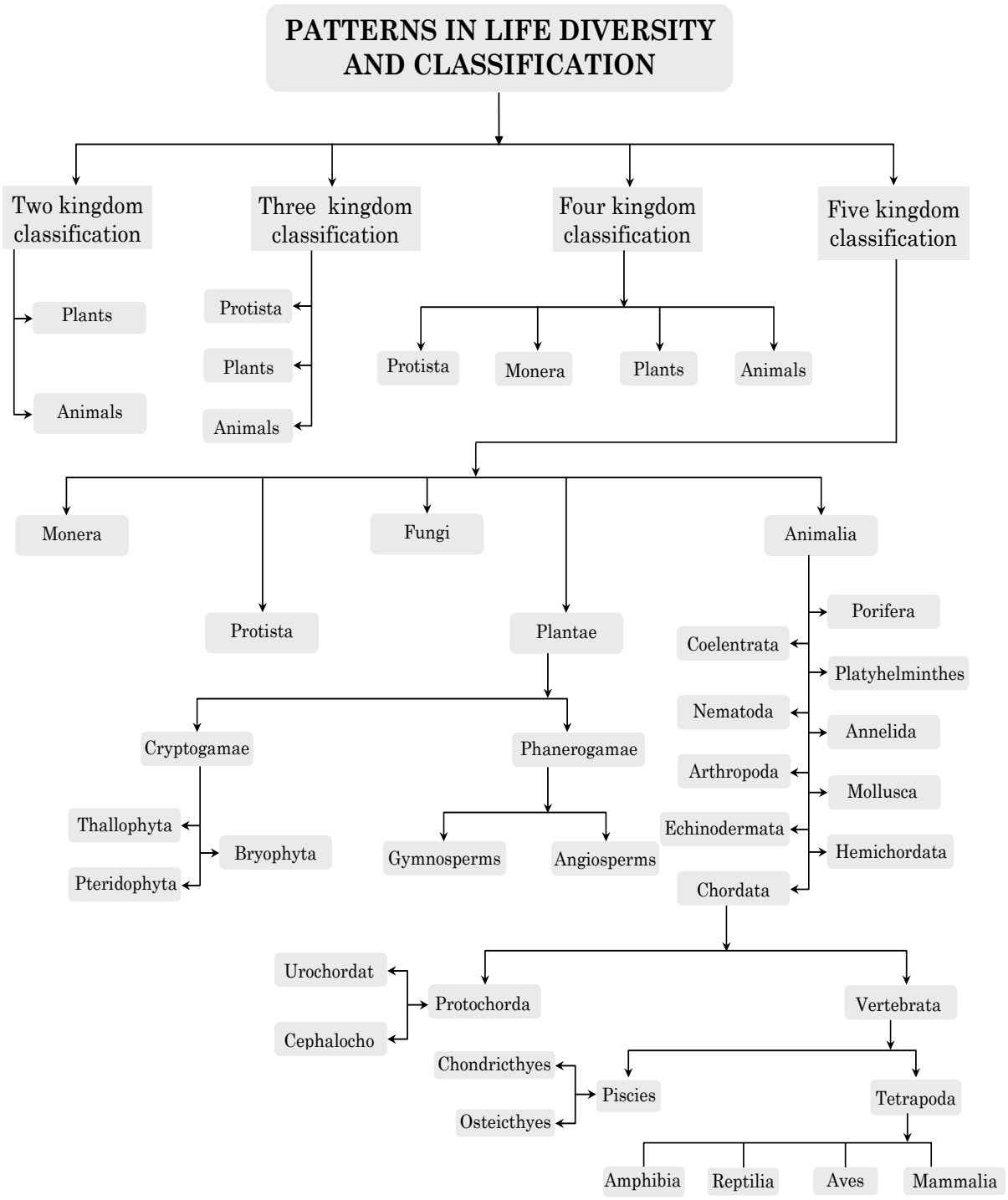
Octopus



Bat

Panthera Tigris

MIND MAP


PATTERNS IN LIFE DIVERSITY AND CLASSIFICATION
Two kingdom classification
Three kingdom classification
Four kingdom classification
Five kingdom classification
Plants
Animals
Protista
Plants
Animals
Protista
Monera
Plants
Animals
Monera
Fungi
Animalia
Protista
Plantae
Porifera
Coelentrata
Platyhelminthes
Nematoda
Annelida
Arthropoda
Mollusca
Echinodermata
Hemichordata
Chordata
Cryptogamae
Phanerogamae
Thallophyta
Bryophyta
Pteridophyta
Gymnosperms
Angiosperms
Urochordat
Cephalocho
Protochorda
Vertebrata
Chondricthyes
Osteicthyes
Piscies
Tetrapoda
Amphibia
Reptilia
Aves
Mammalia

PATTERNS IN LIFE : DIVERSITY AND CLASSIFICATION

Introduction to Biodiversity

The Earth is home to an enormous variety of living organisms. From microscopic bacteria to giant trees and animals, life exists in countless forms and habitats. This great variety of life is known as **biodiversity**.

Biodiversity includes all plants, animals and microorganisms found on Earth along with the ecosystems in which they live. It is essential for maintaining ecological balance and supporting life.

- Every organism performs a specific role in nature.
- Green plants prepare food through photosynthesis and release oxygen into the atmosphere.
- Fungi and bacteria decompose dead plants and animals, converting them into nutrients that enrich the soil.
- Birds, bees and bats help in pollination, while animals help in seed dispersal.

Humans also depend greatly on biodiversity for food, medicines, clothing, shelter and livelihood. Farmers preserve different crop varieties because biodiversity increases resistance against diseases, pests and changing environmental conditions. A rich biodiversity strengthens food security and ecosystem stability.

Thus, all living organisms are interconnected through food chains and ecological relationships.

◈ **Biodiversity :** The term "**biodiversity"** is a concise form of **"biological diversity"** and was coined by **Walter G. Rosen** in 1986.

India as a Biodiversity Hotspot

India is one of the richest countries in biodiversity because of its varied geographical features and climatic conditions. The country has mountains in the north, deserts in the west, plateaus in the south, dense forests in the north-east and long coastal regions. These diverse habitats support a large number of plants and animals.

Some species are found naturally only in a particular region and nowhere else in the world. Such species are known as **endemic species**. Examples include:

- Nilgiri tahr
- Lion-tailed macaque
- Nepenthes khasiana (pitcher plant)
- Neelakurinji

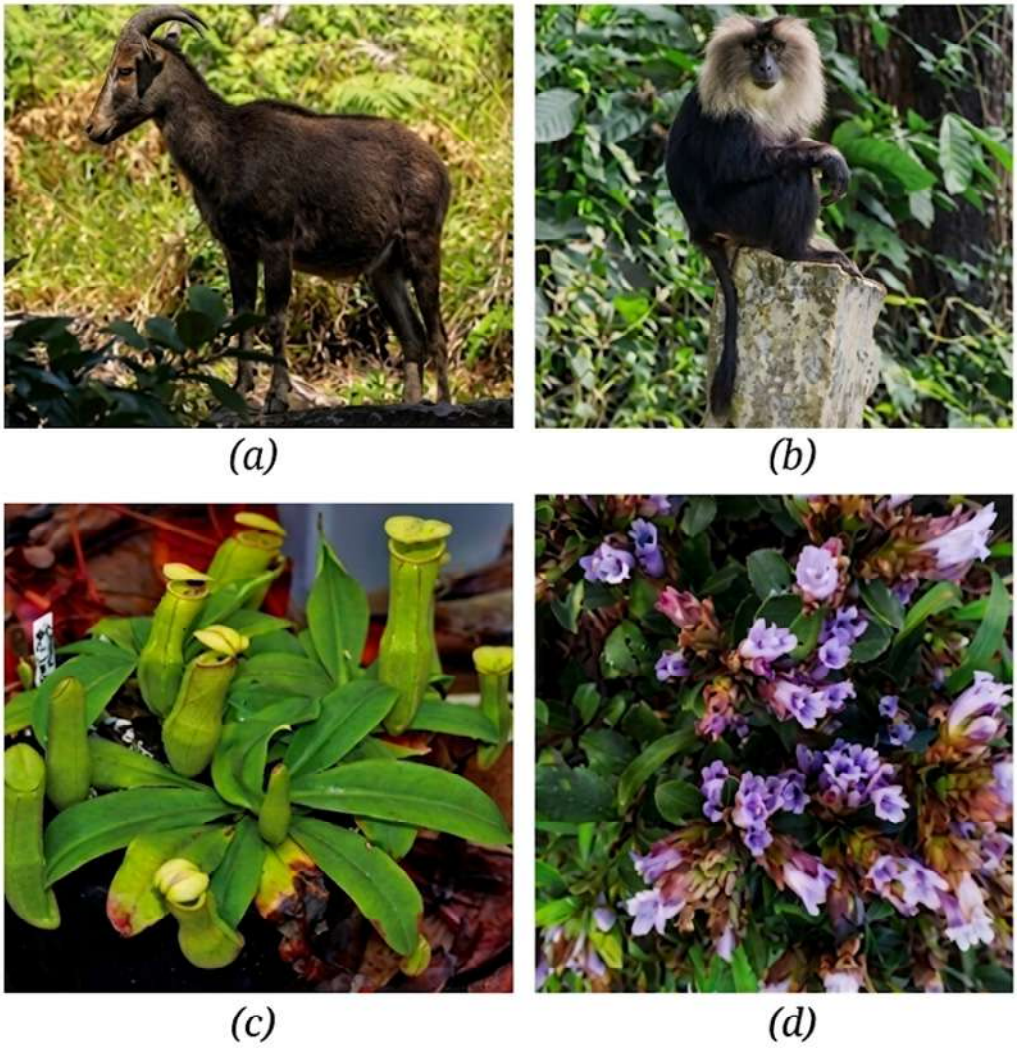


Fig. (a) Nilgiri tahr, (b) Lion-tailed macaque,
(c) Nepenthes Khasiana, and (d) Neelakurinji

Regions that contain a large number of endemic species and face serious habitat loss are called **biodiversity hotspots**. These regions are important for conservation because they contain unique organisms that may become extinct if not protected.

Major biodiversity hotspots of India include:

1. Western Ghats
2. Himalayas
3. Indo-Burma region (North-East India)
4. Sundaland (Nicobar Islands)

◈ How has Biodiversity Evolved?

The biodiversity seen today did not appear suddenly. It developed gradually over millions of years through continuous changes in organisms and their surroundings.

Small variations occur naturally among organisms. Some variations help organisms survive better under environmental changes. Organisms possessing favourable characteristics survive and reproduce successfully. These useful variations accumulate over generations and eventually give rise to new forms of life.

Thus, biodiversity is the result of:

- Continuous changes over time
- Adaptation to environmental conditions
- Evolution from common ancestors

Taxonomy

- The method of arranging organisms into groups on the basis of similarities and differences is called classification.
- (Gr. Taxis-arrangement, nomos-law) The functional branch of biology dealing with the identification, classification and nomenclature of living organisms is called taxonomy. "Carolus Linnaeus" (1707-1778) is known as "Father of taxonomy".

(a) Identification : It is defined as the determination that a particular organism is similar to some other known individual.

(b) Nomenclature : It is the system of naming of any living creature whether it is from a lower phylum or from higher phylum.

(c) Classification : It is the categorization of living creatures according to their peculiar features.

Hierarchy

"Hierarchy" means ordering of living creatures according to their growing similarities. It includes the following pattern of classifying any living one.

The hierarchial order of classifiying organisms is :

Kingdom → Phylum/ Division → Class → Order → Family → Genus → Species

1. **Kingdom** : It is the highest category of taxonomic studies. All animals are included in kingdom Animalia and all plants are included in kingdom Plantae.
2. **Phylum /Division** : Many Classes with some common characters are included in Phylum.
3. **Class** : Similar orders are place together in a class.
4. **Order** : A number of families having common characters are placed in order.
5. **Family** : A number of genera having several common characters form a family.
6. **Genus** : It is a group of closely related species with common ancestory.
7. **Species** : A group of organisms capable of interbreeding to produce offspring.

Ex.1 In the hierarchy of classification, which grouping will have the smallest number of organisms with a maximum of characteristics in common and which will have the largest number of organisms ?

Ans. Species which lie at the bottom of the taxonomic ladder bear the smallest number of organisms and since the members of a species can interbreed among themselves, they possess maximum characteristics in common. In the hierarchy of classification, kingdom is at the top. It bears the largest number of organisms because a kingdom is split into several phyla, classes, orders, families, genera and species, each having their specific characteristics.

COMPETITIVE LEVEL

- Biodiversity "**hotspots**" are areas of the world with a high number of endemic species (that is, species found only in that place).
- **Taxonomists** : Biologists who are specialized in identifying and classifying life on the planet, have named approximately 1.7 million species. About 13,000 more species are added to the list of known organisms. (National Wildlife Magazine)

Nomenclature

- It is the system of naming an individual.
- Nomenclature is done on the basis of a set of rules stated in the ICN i.e. International code of nomenclature.
- **Binomial nomenclature** : Proposed by Carolus Linnaeus in his book Systema Naturae who is also called father of taxonomy.
- According to this system of nomenclature, each animal or plant is given two names, the first one is the generic name and the second one is the name of the species.
- Scientific names are always in *Latin*.
- The first letter of the generic name is always capitalized and that of the species name is written in small letter. For example, the scientific name of frog is *Rana tigrina*, in which *Rana* is the generic name and *tigrina* is the name of the species.

Ex.2 Why is bionomial nomenclature called so ?

Ans. The binomial system of nomenclature gives the scientific name of an organism which consists of two separate words-first one designates generic. Name and the second one designates specific name. Since this system of naming organisms gives two words name to an organism, it is known as a binomial nomenclature.

Ex.3 Define advantage of using scientific name instead of common name ?

Ans. Common names of plants and animals differ from one geographical area to another area but the scientific name is same all over the world, e.g., in Hindi, mango is called as **aam** but in Tamil, it is called as **maangai**. But the scientific name is *Mangifera indica*, which is same all over the world.

COMPETITIVE LEVEL

- In 1980, in the tropical rainforests of Panama, scientists discovered 1,200 species of beetles living in and around just 19 trees and fully 80% of these species were previously unknown to science.
- Approximately half of all synthetic drugs have a natural origin, including 10 of the 25 highest-selling drugs in the US.
- Based on data of recorded extinctions of known species over the past century, scientists estimate that current rates of species extinction are about 100 times higher than long-term average rates based on fossil data.

◈ Classification of Organisms :

The method of arranging organisms into groups on the basis of similarities and differences is called classification.

(a) Significance of Classification :

- It establishes hierarchy of groups of organisms on the basis of their common features.
- It makes the systematic study easier.
- It is essential to understand the inter-relationship amongst different groups of organisms.
- It serves as a base for the development of other biological sciences as well as different fields of applied biology like public health, environment etc.

(b) System of Classification :

(i) Artificial system : Biological classification in early times was based upon single arbitarily chosen character suiting the convenience of taxonomist.

e.g. On the basis of habitat and ability to fly.

(ii) Natural system : It was based on morphological and anatomical similarities and differences.

(iii) Phylogenetic system : It was based on evolutionary sequence as well as genetic relationships amongst the organisms. Charles Darwin showed that living organisms evolved by the process of descent with modifications.

◈ Different Classification Systems :

(a) Two kingdom system : It was given by **Carolus Linnaeus** in 1758. Organisms were divided into Plantae kingdom and Animalia kingdom. Fungi, bacteria and *Euglena* could not find an appropriate position.

(b) Three kingdom system : It was given by **Ernst Haeckel** (1894). In this kingdom Protista was also included along with plant kingdom and animal kingdom.

(c) Four kingdom system : It was given by **Copeland**. kingdom Monera was also included in this system of classification.

(d) Five kingdom classification : This scheme was proposed by **R.H. Whittaker** in 1969. Fungi could not find a suitable place in earlier system of classifications and thus Whittaker's five kingdom theory was found to be most favoured amongst most biologists.

This classification includes :

(i) Monera (ii) Protista (iii) Fungi (iv) Plantae (v) Animalia

Ex.4 Why do we classify organisms ?

Ans. There is vast number of living organisms in this biosphere and they have a great diversity in shape, size and forms. It is practically not possible to examine and study every organism separately at individual level. It is, therefore, advisable to study the diversity of organism by classifying them in orderly manner.

◈ Basis of Five Kingdom Classification :

- Complexity of structure
- Mode of nutrition
- Level of organization

(i) Kingdom Monera : Unicellular, prokaryotic, microscopic, most ancient, can live in deep oceans, hot springs, deserts, high salt concentrations etc. They include bacteria, filamentous and photosynthetic blue green algae (Cyanobacteria) etc.

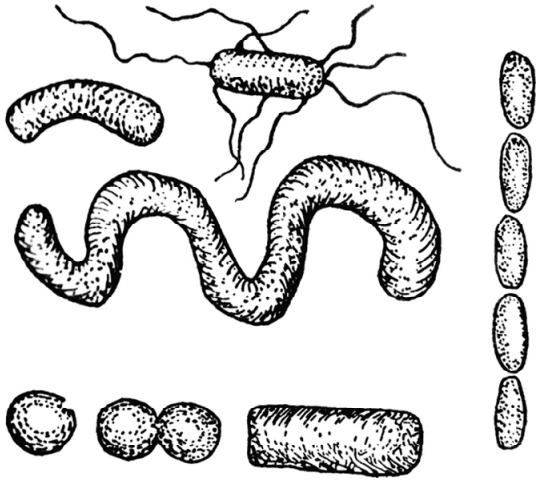
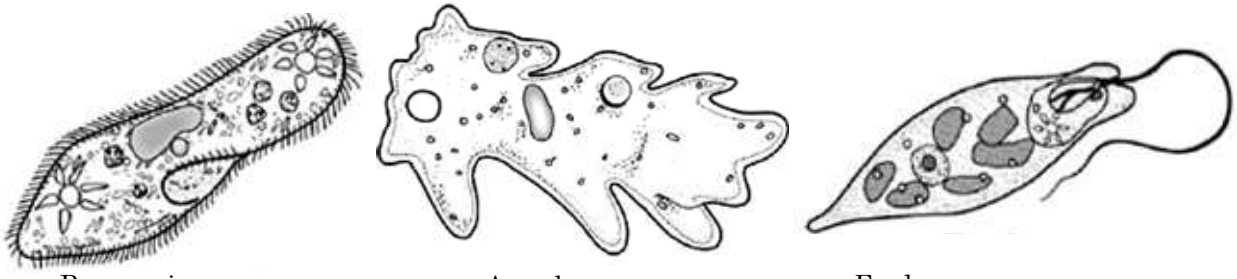


Fig. : Different Types of Bacteria

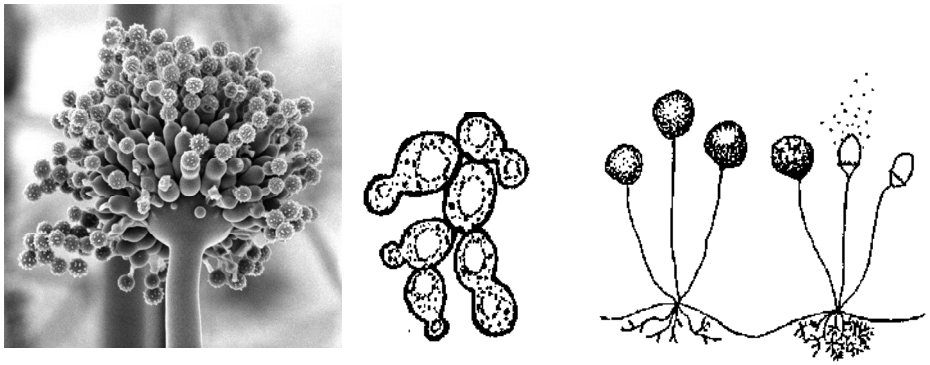
(ii) Kingdom Protista : Unicellular, colonial, eukaryotic. They include photosynthetic algae, decomposers (slime moulds) and protozoa (predators) etc.



Paramecium Amoeba Euglena

Fig. : Protista

(iii) Kingdom Fungi : Unicellular or multicellular eukaryotic organisms. They are heterotrophic, parasitic or saprotrophic. They include ascomycetes and basidiomycetes.



Aspergillus Yeast Spore Formation

Fig. : Fungi

(iv) Kingdom Plantae : They are multicellular, eukaryotic, autotrophic (photosynthetic), some are heterotrophic and parasitic.

They include photosynthetic algae, green plants etc.

(v) Kingdom Animalia : Multicellular, eukaryotic, heterotrophic.

Ex.5 Give three examples of the range of variations that you see in life forms around you.

Ans. The three examples are :

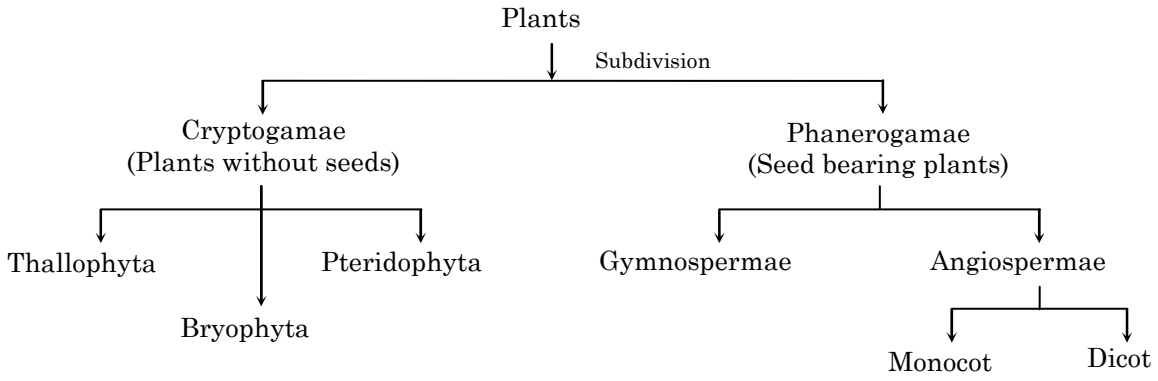
(i) The living organisms vary in size form a few micrometers (e.g., microscopic bacteria) to more than 30 metres long (e.g., blue whale) and more than 100 metres tall (e.g., red wood trees of California).

(ii) The living organism vary in longevity from a few days (like mosquitoes) to several thousand years (pine trees).

(iii) The living organisms range from colourless or transparent to brightly coloured birds and flowers.

◈ Plant Classification :

Plant kingdom was divided into two sub kingdoms by Eichler.



A. Sub kingdom cryptogamae :

(Cryptos = hidden gamous = marriage): It includes lower plants which do not bear flower or seeds.

(a) Division Thallophyta :

- It comprises of algae.
- They are most primitive and simple kind of plants in which plant body is not differentiated into root, stem and leaves. Xylem and phloem are absent. i.e. vascular system is absent.
- Reproductive organs are single–celled and there is no embryo formation after fertilization.
- These are aquatic or terrestrial, fresh water or marine. Algae are photosynthetic containing various pigments like chlorophyll, carotenoids, xanthophylls etc. These can be unicellular, colonial, filamentous and multicellular.
- Cell wall is made up of cellulose and stored food is starch. E.g. Green algae (*Ulothrix*, *Chara, Cladophora, Ulva, Spirogyra*), Brown algae, Red algae etc.

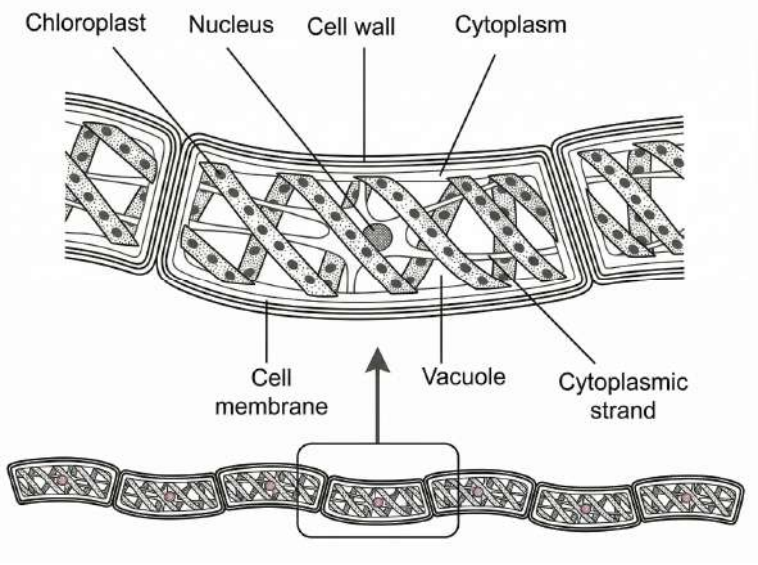


Fig. : Spirogyra

(b) Division Bryophyta :

- The division Bryophyta (Greek word *Bryon* **=** moss **;** *phyton* = plant) includes the simplest and primitive plant.
- Xylem and phloem are absent (non – vascular). Plant body is differentiated into leaf and stem like structures and root like structures (**rhizoids).**
- They are also known as amphibian of plant kingdom because they require water for the completion of their life cycle. An embryo is formed upon fertilization. Sporophyte lives as a parasite over gametophyte. Sex organs are multicellular. The male sex organs are **antheridia** and female sex organs are **archegonia.**
- E.g. Liverworts (*Riccia*, *Marchantia*), hornworts (*Anthoceros*) and Mosses (*Funaria*, *Polytrichum*).

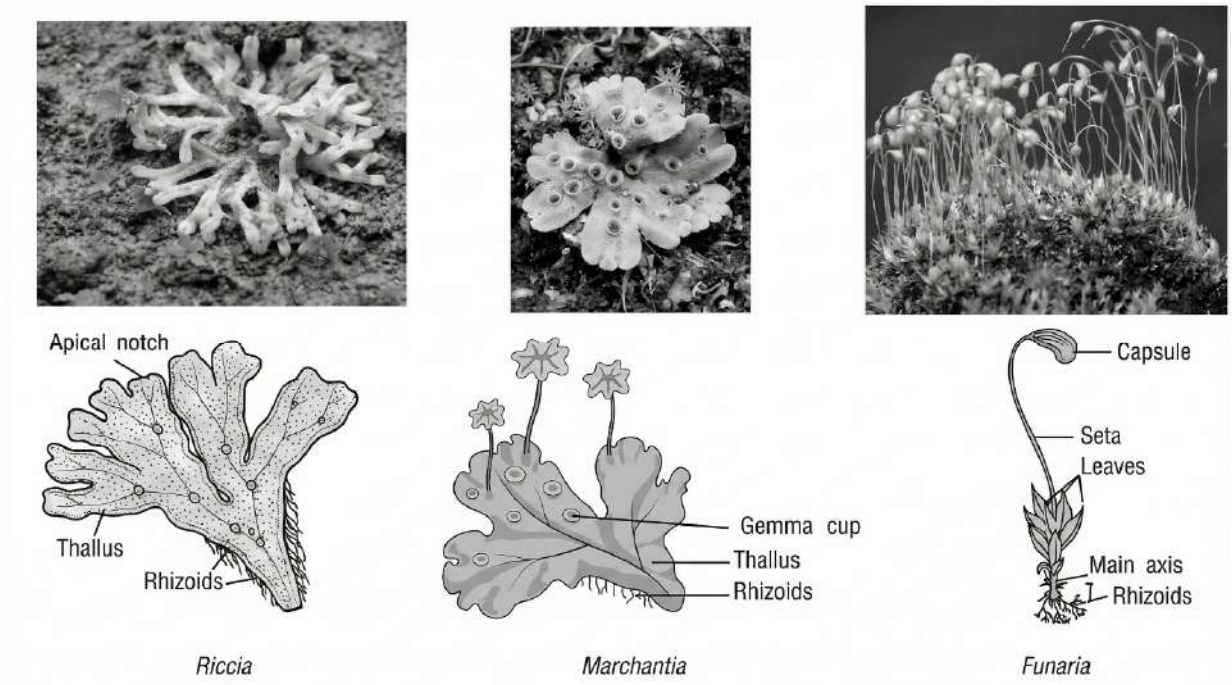


Fig. : Bryophytes

(c) Division Pteridophyta

- Plant body is differentiated into root, stem and leaf. Xylem and phloem are present.
- These plants have no flowers and do not produce seeds. Sex organs are multicellular and jacketed by sterile cells.
- Club mosses [*Selaginella*, *Lycopodium* ("ground pine")], horsetails (*Equisetum)* and ferns (*Marsilea).*

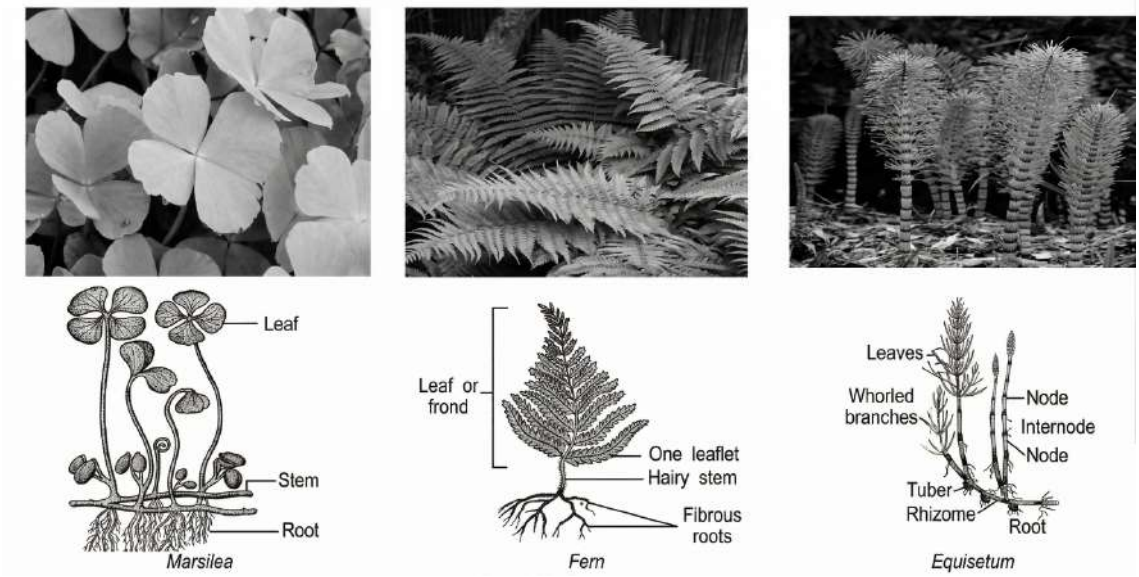


Fig. : Pteridophyta

B. Sub kingdom : Phanerogamae

(Phaneros = visible : gamous = marriage.) : This is the highest group of plants which includes seed bearing plants. They are of two types :

(a) Gymnosperms :

- The seeds produced by these plants are naked and are not enclosed within fruits. They have well developed vascular system but xylem lacks vessels and phloem lacks companion cells.
- Plants are commonly tall trees or shrubs. The reproductive organs are represented by unisexual cones. Often both cones are present on the different plant except pine (monoecious). E.g. : *Cycas* , *Pinus*, *Ginkgo*

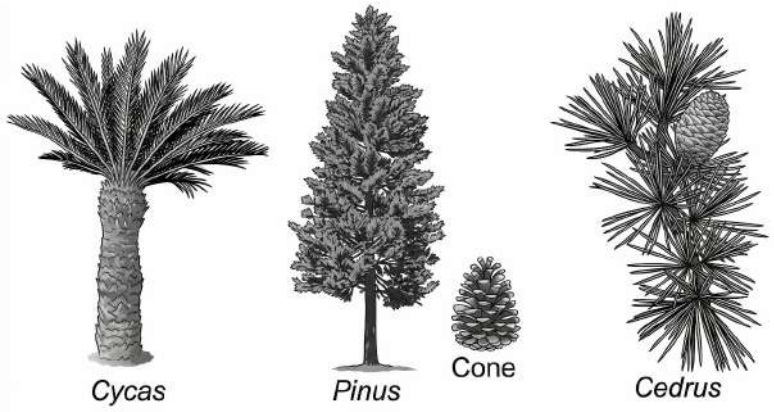


Fig. : Gymnosperms

(b) Angiosperms :

- Angiosperms are highly evolved plants and they produce seeds that are enclosed within the fruit. The reproductive organs are aggregated in a flower.
- These are represented by trees, herbs, shrubs. They have highly developed vascular system. Seeds remain enclosed in ovary. Plant embryos in seeds have structures called cotyledons. Cotyledones are called "seed leaves".
- It is divided into two classes on the basis of number of cotyledons.

(i) Dicotyledonae :

- The seeds produced by these plants have embryos with two fleshy leaves, the cotyledons.
- Their leaves have reticulate venation, with a network of veins. The root system has a prominent tap root.
- E.g. : Pea (*Pisum sativum*), potato (*Solanum tuberosum*), sunflower (*Helianthus annuus*), rose (*Rosa indica*), banyan (*Ficus religiosa*), neem (*Melia indica*), apple (*Malus silvestris*).

(ii) Monocotyledonae :

- The seeds of these plants have only one cotyledon.
- Their leaves have parallel venation. The root system consists of fibrous roots.
- The vascular bundles are scattered and closed (i.e. lack cambium). Secondary growth does not occur.
- **Examples :** Maize (*Zea mays*), wheat (*Triticum vulgare*), rice (*Oryza sativa*), onion (*Allium cepa*), sugarcane (*Saccharum officinarum*), barley (*Hordeum vulgare*), banana (*Pandanus*), Coconut and grasses.

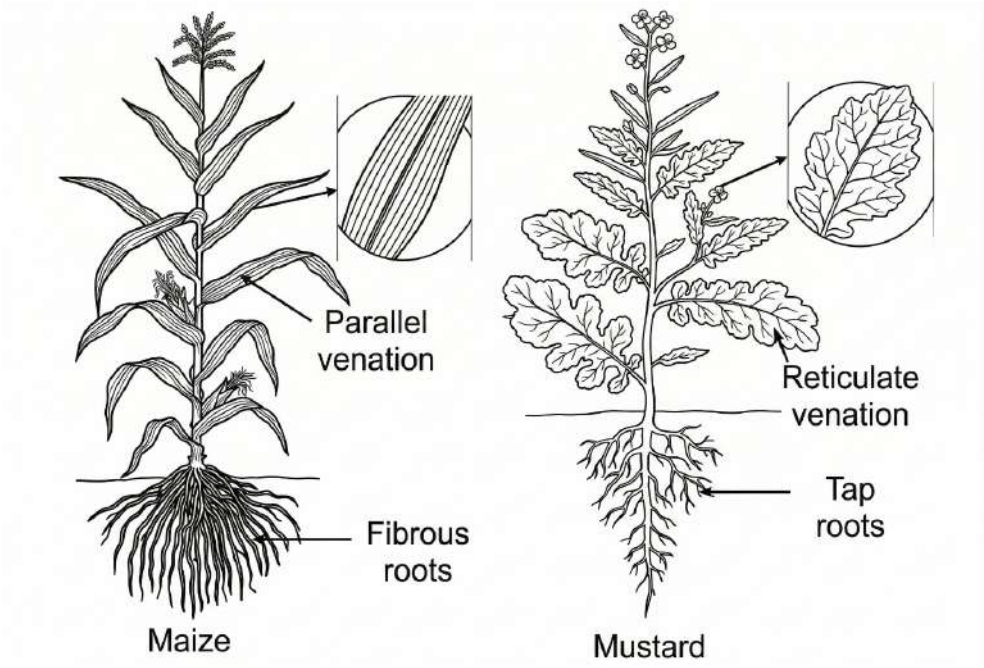


Fig. : Monocot plant and Dicot Plant

◈ Animal Kingdom :

Basis of Classification :

- Organization and differentiation of cells to form tissues and organs.
- Body symmetry.
- Formation of body cavities and blood vascular system.
- Features of embryonic development.

(a) Level of organization in living beings : It is structural differentiation of animal body.

(i) Cellular level of organization : Tissues do not differentiate. Different types of cells are present, e.g., Porifera (sponges).

(ii) Tissue level of organization : Multicellular body cells organised into tissues but organs are absent e.g., Coelenterates.

(iii) Organ level of organization : Cells are organized into tissues and tissues into organ but organ systems are absent e.g., Platyhelminthes.

(iv) Organ system level of organization : Cells are organised into tissues, tissues into organs and organ into organ systems e.g., Annelids, arthropods, chordates.

(b) Body symmetry : It is similarity in arrangement of parts. Symmetry is of two types, radial and bilateral.

(i) Radial symmetry : The body is cylindrical or discoid when similar parts occur all around the central axis. Any vertical plane passing through the central axis will divide the body into two equal halves, e.g., ctenophore, coelenterates and echinodermates.

(ii) Bilateral symmetry : The body has a head. Organs and limbs are paired. They are arranged laterally. Body is divisible into two equal halves by only one plane (mid-sagittal plane). Bilateral symmetry is found in Platyhelminthes, Nematoda, Annelida, Mollusca, Arthropoda and Chordata.

COMPETITIVE LEVEL

- Carl Woese (1977) modified the five kingdom classification by dividing the Monera into Archaebacteria (or Archaea) and Eubacteria (True Bacteria). This is known as six kingdom classification.
- **Cephalization :** It is development of head in the anterior part of the animal body.

(c) Germ or germinal layers :

- A germ layer is group of cells in an embryo that interact with each other as the embryo develop and contribute to formation of all organs and tissue.
- Germinal layers can be two or three in number. On this basis of the same, the animals are of two types, diploblastic and triploblastic.

(i) Diploblastic animals : Animals having two germinal layers, outer ectoderm and inner endoderm. Mesoderm is absent e.g. Porifera, Coelenterata.

(ii) Triploblastic animals : Animals having three germinal layers-outer ectoderm, middle mesoderm and inner endoderm, e.g. Platyhelminthes to chordates.

(d) Coelom (body cavity): It is mesoderm lined fluid filled space that occurs between alimentary canal and body wall. Depending upon the absence or presence of coelom, animals are of three types- acoelomate, pseudocoelomate and eucoelomates.

(i) Acoelomate : Coelom is absent in Porifera, Coelenterata, Platyhelminthes. InPlatyhelminthes, a mesoderm is present but it does not form cavity.

(ii) Pseudocoelomate : A cavity called pseudocoelom is present which is not lined by mesoderm. It is generally endodermal in origin. Mesoderm occurs but forms small separate pouches e.g., Aschelminthes.

(iii) Coelomate or Eucoelomate : A true coelom lined by mesoderm is present. E.g. Annelids to chordates.

(e) Notochord is a mesodermally derived rod-like structure formed on the dorsal side during embryonic development in some animals. It is solid rod of polygonal shaped cells, present on dorsal side between alimentary canal and nervous system in some animals.

On the basis of presence or absence of notochord animals are of two types :

(i) Chordates : Animals with notochord are called chordates, e.g. Amphibians, Reptiles, Aves, Mammals.

(ii) Non chordates : Animals which do not form notochord are called non-chordates, e.g., Porifera to echinoderms.

COMPETITIVE LEVEL

- In some animals, the body is externally and internally divided into segments with a serial repetition of at least some organs. For example, in earthworm, this phenomena is known as metamerism.
- Annelids are the first animal to develop true body cavity or coelom.
- Cnidaria is the first phylum to be organized at tissue level organization and Platyhelminthes is the first to be organized at organ system level of organization but it lacks well developed organs due to absence of coelom.
- On the basis of origin, true coelom is of two types, schizocoelom and enterocoelom.

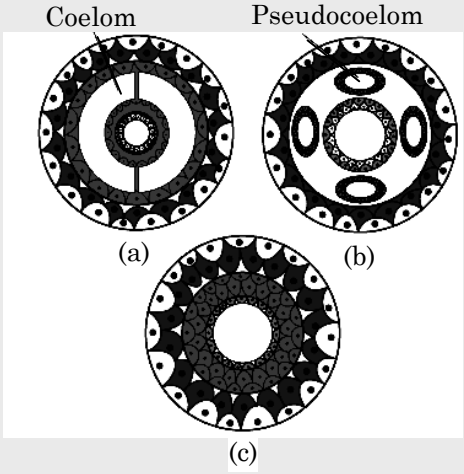


Fig. : Diagrammatic sectional view of :
(a) Coelomate
(b) Pseudocoelomate
(c) Acoelomate

◈ Non Chordates :

(a) Phylum Porifera :

- Sessile (stalk-less) and marine (*Sycon*) except one group that lives in fresh water (*Spongilla*).
- Simplest multicellular, that have cellular level of organization.
- Hard exoskeleton present.
- Sponges are mostly asymmetrical. Body is perforated by numerous pores, the ostia that open into a canal system having canals and chambers lined with collared flagellated cells or choanocytes.
- **Examples :** *Sycon*, *Euplectelea*, (Venus flower basket), *Spongilla* (Fresh water sponge).

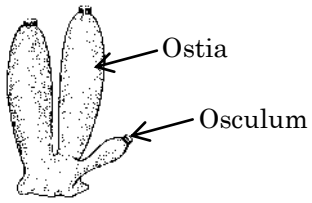


Fig. : Sycon

Fig. : Euplectelea

(b) Phylum : Cnidaria (Coelenterata) :

- All members are aquatic, mostly marine, a few such as *Hydra* are fresh water solitary or colonial forms.
- Cnidarians or coelenterates are multicellular, diploblastic animals with tissue grade of organization. Their body shows radial symmetry.
- **Examples :** *Hydra*, *Obelia* (sea fur), *Aurelia* (jelly fish), *Metridium* (sea anemone).

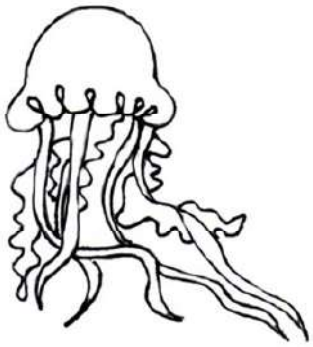


Fig. : Aurelia (Jelly fish)

COMPETITIVE LEVEL

- Body of poriferans may be vase-like, rounded, sac-like and branched. Body contains a cavity called as spongocoel.
- Coelenterates possess specialized cells (cnidoblasts or cnidocytes) bearing stinging capsules called nematocysts present on tentacles and body. Nematocysts serve the functions of paralysing the prey by injecting poison or to hold the prey.
- Coelenterates exhibit the phenomenon of polymorphism means body shows two main forms, the polyps and the medusae.

◈ **Phylum Ctenophora :**

- These organisms are marine, solitary and free swimming with radially symmetrical and transparent body.
- They possess comb plates that are ciliated which help in locomotion. They also possess tentacles.
- **Examples :** *Cestum, Ctenoplana, Pleurobrachia.*

◈ **Bioluminescence is** the property of a living organism to emit light which is found in ctenophores.

(c) Phylum Platyhelminthes :

- These animals are bilaterally symmetrical, acoelomate and dorsoventrally flattened, hence are also called as flatworm. Body is thin, soft, leaf-like or ribbon-like.
- Triploblastic animals with organs level of organization. Digestive cavity is with a single opening, the mouth. Suckers and hooks are usually present.
- **Examples** : *Dugesia* (*Planaria*), *Fasciola* (liver fluke), *Schistosoma* (blood fluke), *Taenia solium* (pork tape worm).

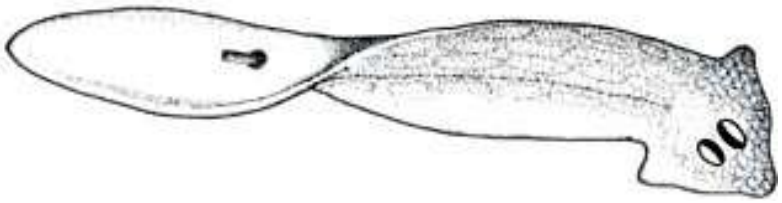


Fig. : Planaria

(d) Phylum Aschelminthes or Nematoda :

- They are parasitic or free-living with complete alimentary canal.
- They are triploblastic, unsegmented and show bilateral symmetry. Body cavity is not a true coelom but a pseudocelom.
- Sexes are separate (sexual dimorphism).
- **Examples** : *Ascaris* (Round worm), *Enterobius* (Pin worm), *Wuchereria* (filarial worm).

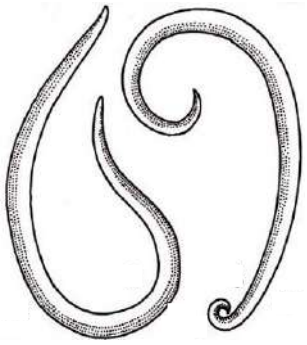


Fig. : Ascaris

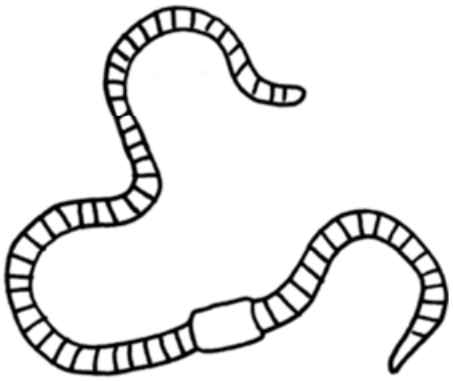


Fig. : Earthworm

(e) Phylum Annelida :

- These organisms occur in moist soil, fresh water and sea. These are elongated, with metamerically segmented body and bilateral symmetry.
- First animals with true body cavity. Body bears lateral appendages for locomotion in the form of chitinous setae or parapodia.
- **Examples :** *Nereis* (sand worm) *Aphrodite* (sea mouse), *Pheretima* (earthworm), *Hirudinaria* (cattle leech)

(f) Phylum Arthropoda :

- Triploblastic organisms with bilaterally symmetry and segmentation.
- Body segments are grouped into two regions-cephalothorax (head and thorax together) and abdomen or three regions-head, thorax and abdomen.
- Each body segment usually bears paired lateral and jointed legs or appendages.
- Coelom is reduced and body is covered with a thick chitinous covering.
- Respiration through general body surface, by gills, air tubes (tracheae) or book-lungs.
- **Example :** *Palemon* (prawn), *Daphnia* (water flea), *Limulus* (king crab), *Palamnaeus* (scorpion)

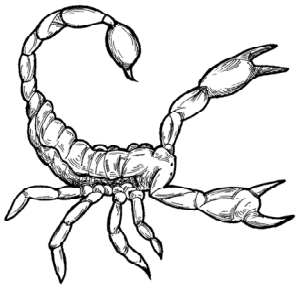


Fig. : Scorpian

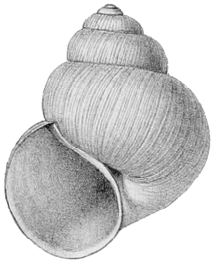


Fig. : Pila

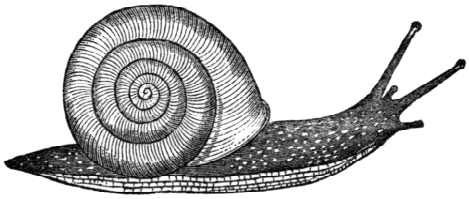


Fig. : Snail

COMPETITIVE LEVEL

- In flatworms, circulatory system, respiratory system and skeleton are absent. Excretory system occurs through specialized cells called flame cells.
- Planaria is hermaphrodite animal (an animal that contains both male and female reproductive organs).
- Arthropoda is largest phylum of animal kingdom.

(g) Phylum Mollusca :

- They have soft, unsegmented body. These are bilaterally symmetrical, triploblastic, Coelomate and having organs system level of organization.
- Body is divided into three regions head visceral mass and ventral foot. Outer surface is covered by a hard calcareous shell.
- Respiration is by gills called ctenidia. The sexes are usually separate.
- **Examples :** *Chiton*, *Pila* (Snail), *Unio* (Fresh water mussel), *Octopus* (Devil fish).

(h) Phylum Echinodermata :

- Radially symmetrical, triploblastic, coelomate animal with organ system level of organization.
- They are marine, gregarious (live in groups) and free-living animals.
- Shape may be star-like, spherical or elongate. Body surface is covered all over by calcareous spines.
- Tube feet for locomotion. These are unsegmented.
- Body cavity is modified into a water-vascular system or ambulacral system with tube like outward extension for locomotion, called tube feet.
- **Examples :** *Asterias* (star fish), *Echinus* (sea urchin), *Holothuria* (sea cucumber), *Antedon* (feather star)

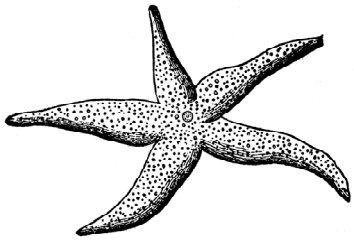


Fig. : Starfish

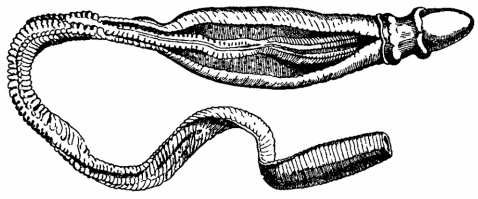


Fig. : Balanoglosus

(i) Phylum Protochordata :

- Triploblastic, coelomate animal with organ system level of organization.
- They are placed in between nonchordates and chordates as they possess some characters of both.
- They include worm like, unsegemented, bilaterally symmetrical animals which are exclusively marine.
- Their body is divided into three regions proboscis, collar and trunk.
- They do not possess notochord, which is a flexible, rod like structure running through the length of the body, above alimentary canal.
- They possess gill slit or gill cleft which is meant for respiration.
- Nerve cord in collar region is present but it is not a true dorsal nerve cord.
- **Examples :** *Balanoglossus* (tongue worm), *Saccoglossus*.

COMPETITIVE LEVEL

- Dorsal nerve chord is solid strand of nervous tissue forming a part of central nervous system.
- Difference between nerve chord and notochord is that notochord leads to form vertebral column while nerve chord leads to form spinal cord.

(j) Vertebrata :

- They are advanced animals, having a cranium (brain box) around the brain. Nervous system is well developed.
- Notochord is replaced by a vertebral column (backbone) in the adults. Endoskeleton is highly developed.

- There are two pairs of limbs or appendages.
- Head is well differentiated.
- The heart is situated ventrally. The circulatory system is closed consisting of blood vascular system and lymphatic system. Red coloured pigment haemoglobin is present in red blood corpuscles.
- Respiratory organs may be gills (in aquatic animals), skin, buccopharyngeal cavity (in amphibians) or lungs (in land animals).
- Excretion occurs through kidneys.
- Sexes are separate.

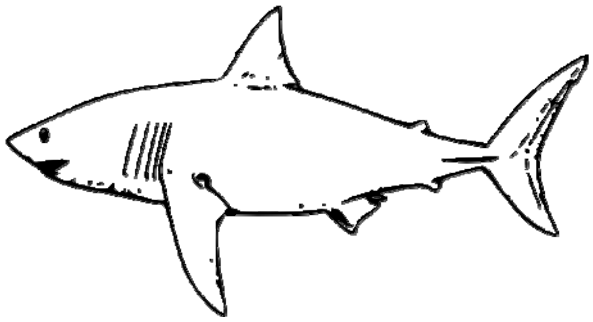


Fig. : Shark

(i) Super class Pisces :

- True fishes are included in this class which respire through gills.
- Their body is stream lined and covered by scales / plates.
- They have muscular tail and fins for movement.
- Endo skeleton is either made up of cartilage or bone.
- They are unisexual and lay eggs.
- They are cold blooded with two chambered heart.

COMPETITIVE LEVEL

S.No.	CHONDRICHTHYES	OSTEICHTHYES
1.	Mouth ventral.	Mouth terminal.
2.	Tail fin asymmetrical (heterocercal).	Tail fin symmetrical (homocercal).
3.	5-7 pairs of gills are present.	4-5 pairs of gills are present.
4.	Gills are naked.	Gills are covered by operculum.
5.	Cloacal aperture is present.	Anus and urinogenital apertures are separate.
6.	No swim bladder.	Swim bladder usually present.
7.	Fertilization is internal.	Fertilization is external.
8.	e.g. Scoliodon – Dog fish, Trygon - Sting ray, Torpedo - Electric ray, Rhineodon - Whale shark.	e.g. Labeo rohita - Rohu or Indian carp, Anabas - Climbing perch, Caulophyryne jordani- Angler fish, Hippocampus - Sea horse, Pterois valitans -Lion fish, Exocoetus - Flying fish.

(ii) Class Amphibia :

(Greek, amphi = both, bios = life, the vertebrates leading two lives / dual life)

- They can live on land as well as in water.
- Skin is smooth or rough, rich in glands which keep it moist and with pigmented cells, i.e. chromatophores.
- Respiration occurs by lungs, skin or buccal lining. Gills are present at least during larval stage for respiration.
- Heart is three chambered with two auricles and a ventricle. Red blood corpuscles are large, biconvex, oval and nucleated. Brain is not much developed. Cranial nerves are 10 pairs.
- Eggs with gelatinous covering and are usually laid in water. Fertilization is external.
- Development is indirect with a tadpole larva which undergoes metamorphosis to become adult.
- **Example :** Salamanders, frogs and toads, *Necturus* (mud puppy), *Triturus* (newt), *Rana* (frog), *Bufo* (toad).

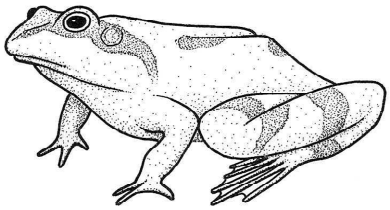


Fig. : Frog

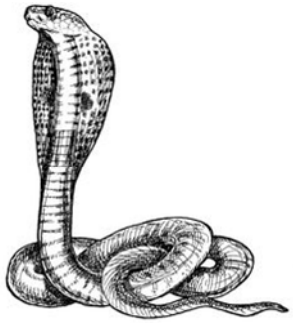


Fig. : Snake

(iii) Class Reptilia :

- In these animal two pairs of pentadactyl limbs are present, but in snakes limbs are reduced or absent.
- Body is covered with epidermal horny scales. Skin is dry, impermeable and devoid of glands.
- Respiration takes place by lungs only. Gills are absent.
- Heart is incompletely four – chambered, having two auricles and incompletely divided ventricles. In crocodile, heart is completely four chambered.
- Sexes are separate. Fertilization is internal.
- **Examples:** *Testudo* (tortoise), *Chelone* (turtle) *Draco* (flying lizard), Chameleon, *Hemidactylus* (wall lizard), *Naja* (cobra) etc.

(iv) Class Aves : (L. Aves = birds)

- The birds are described as 'feathered reptiles' that have developed the power of flight.
- The body is covered with soft feathers called plumage. The body is divisible into head, neck, trunk and tail.
- There are two pairs of limbs. The fore limbs are modified to form wings or are reduced. Hind limbs are strongly developed for perching, walking.
- Endoskeleton is light. The bones have air cavities. This makes the bird light.

- Respiration is by lungs only. Lungs have additional bag like membranous extensions called as air sacs.
- Heart is completely four chambered.
- Sexes are separate.
- Birds are oviparous, i.e. egg laying.
- **Examples :** *Columba* (pigeon), *Pavo* (peacock), *Corvus* (crow), *Passer* (sparrow). *Struthio camelus* (ostrich), white stork (Ciconia ciconia), tufted duck (Aythya fligula). *Kiwi* and *Penguin* are flightless birds.

Fig. : Sparrow

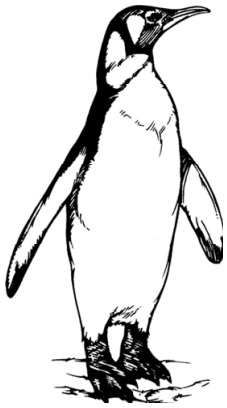


Fig. : Penguin

COMPETITIVE LEVEL

- Body is without scales. Endoskeleton is mostly bony. Notochord does not persist in adults.
- Head and trunk are distinct. Neck and tail may or may not be present.
- They are tetrapods (four – limbed) having digits without nails.
- Excrete either ammonia (by tadpole) or urea (by adults).
- Sexes are separate, i.e., dioecious. Male is without copulatory organ.
- Jaws are modified to form a strong beak. Teeth are absent.
- High degree of parental care is exhibited. There is no larval stage in development.
- Fertilization is internal. Fertilized eggs are laid with a yolk (stored food) and with a hard calcareous shell.

(v) Class Mammalia :

- Mammalia is the most evolved group of organisms and are found in diverse habitats ranging from deserts, polar ice caps, oceans, mountains, forests and grasslands.
- Skin is covered with an exoskeleton of hair. Hair are provided with sweat glands which help in the regulation of body temperature. In aquatic mammals, hair being negligible, the subcutaneous layer of fats provides insulation.
- Mammals have two pairs of pentadactyl limbs.

- The body cavity is unequally divided into two parts by a muscular partition called as diaphragm.
- Eyes are provided with movable lids.
- Ears have fleshy external ears or pinnae.
- Teeth are embedded in sockets (thecodont). Two sets of teeth develop in the life time of a mammal, milk teeth and permanent teeth (diphyodont). Teeth are of different types (heterodont).
- Respiration occurs by lungs. Heart is four chambered. R.B.Cs are non nucleated and usually circular.
- Sexes are separate. Gonads are paired. Testes lie commonly in the scrotal sacs outside the abdomen.
- Fertilization is internal. Eggs are small, microscopic without shells and are retained in uterus of female for development.
- Mammals are divided into three main groups :

(1) Egg-laying mammals (monotremes) : These mammals show characters of both reptiles and mammals. They lay hard shelled eggs (oviparous). E.g., Spiny ant eater, duck billed platypus

(2) Marsupial mammals (pouched mammals) :

- Pouched or marsupial mammals (Latin marsupium = pouch) They are viviparous.
- The young ones, when born, are carried in pouch called marsupium present on the mother's abdomen.
- In the pouch, they feed on the mother milk. Example : Kangaroo (*Macropus*), kola bear.

(3) Placental mammals (true mammals) : These mammals with true placenta.

- The embryo is retained in the uterus.
- These are the very successful group of land animals, occurring in diverse climatic conditions.
- Example : Bat, lion, tiger, camel, giraffe, whale, dolphin, monkey, humans etc.

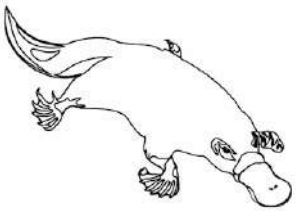


Fig. : Platypus

Fig. : Kangaroo

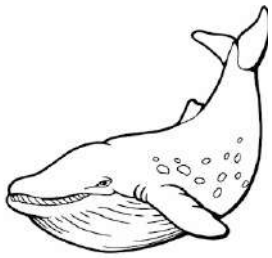


Fig. : Whale

Note : They are named mammals as all of them possess mammary glands (milk producing glands). Mammals are the only animals which feed their young ones with milk.

Ex.6 Will advanced organisms be the same as complex organisms? Why?

Ans. The complexity of body design will increase over evolutionary time. This is the process by which new modifications occur in the organisms. So, in this way we can say that advanced organisms will be complex as compared to complex organisms.

Ex.7 Define the main characteristics of Vertebrates.

Ans. These animals have a true vertebral column and internal skeleton, which allows a completely different distribution of muscle attachment points to be used for movement.Vertebrates are bilaterally symmetrical, triploblastic, coelomic and segmented, with complex differentiation of body tissues and organs.

Ex.8 Bat can fly still it is placed in mammals. Why?

Ans. Bat can fly still it is placed in mammals because:

(i) Gives birth to the young ones.

(ii) Presence of mammary glands.

(iii) Presence of external ears.

(iv) Presence of teeth in beak.

(v) Presence of hair on body.

(vi) Presence of diaphragm between thoracic and abdominal cavity.

Ex.9 Why are whales not grouped in fishes ?

Ans. Whales belong to Class Mammalia while fishes are included in class Pisces. Both are aquatic animals but differ in several basic characteris, such as :

(a) Whales are warm-blooded animals while fishes are cold-blooded.

(b) Whales respire through lungs while fishes respire through gills.

(c) Whales possess mammary glands which are absent in fishes.

(d) Whales have four-chambered heart, while fishes have a two chambered heart.

Fossils as Evidence

Fossils are preserved remains or impressions of ancient organisms found in rocks.

Importance of Fossils

- Provide evidence of evolution
- Show changes in organisms over time
- Help scientists study extinct organisms

Older rock layers generally contain simpler organisms, while newer layers contain more complex organisms.

Biodiversity Under Threat

Human activities are destroying biodiversity rapidly.

Causes of Biodiversity Loss

1. Deforestation
2. Pollution
3. Climate change
4. Overuse of natural resources
5. Habitat destruction

The extinction of one species affects many others connected to it through food chains and ecological interactions.

◈ Need for Conservation

Conservation of biodiversity is necessary for:

- Ecological balance
- Sustainable development
- Survival of future generations

Viruses

Biological diversity includes a vast range of organisms differing in size, structure, complexity, and mode of life. Most living organisms are composed of one or more cells and are therefore classified as **cellular organisms**. However, certain biological entities lack cellular organization. These are termed **acellular entities**.

Among acellular entities, **viruses** occupy a unique and significant position. They do not possess a cellular structure, yet they contain genetic material and exhibit certain characteristics associated with living organisms. Because of this dual nature, viruses are often regarded as existing at the boundary between living and non-living matter.

A virus is an ultramicroscopic, infectious acellular particle composed of genetic material enclosed within a protein coat. It can replicate only inside a living host cell by utilizing the host's metabolic machinery.

Viruses are described as **obligate intracellular parasites**, meaning that they are completely dependent on living cells for reproduction and survival.

Outside the host cell, viruses remain inert and show no metabolic activity.

◈ Discovery of Viruses

The discovery of viruses dates back to the late nineteenth century. In 1892, the Russian scientist **Dmitri Ivanovsky** observed that the infectious agent causing tobacco mosaic disease could pass through filters that retained bacteria.

Later, in 1898, the Dutch microbiologist **Martinus Beijerinck** concluded that the agent was a new type of infectious entity and named it "virus."

This discovery marked the beginning of virology, the scientific study of viruses.

◈ Structural Organization of Viruses

Viruses are extremely small, generally ranging from 20 to 300 nanometres in size, and can be observed only under an electron microscope.

A typical virus consists of the following components :

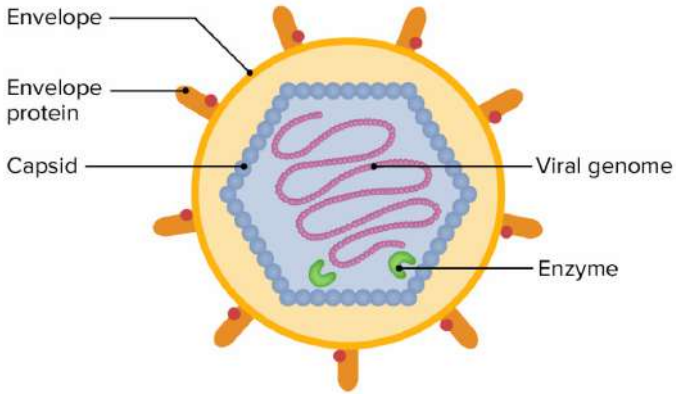
1. **Genetic Material :** The core of a virus contains nucleic acid, which may be either :
   - DNA (Deoxyribonucleic acid), or
   - RNA (Ribonucleic acid)

   Unlike cellular organisms, a virus contains only one type of nucleic acid.

   The genetic material carries the instructions necessary for viral replication.
2. **Capsid (Protein Coat) :** The nucleic acid is surrounded by a protective protein covering known as the **capsid**. The capsid is made up of protein subunits called capsomeres.

   Functions of the capsid include :
   - Protecting the genetic material.
   - Helping the virus attach to the host cell.
   - Determining the shape of the virus.

3. **Envelope (in Some Viruses) :** Certain viruses possess an outer lipid membrane called an **envelope**, derived from the host cell membrane. This envelope contains specialized proteins that assist in entering host cells.



◈ Types and Shapes of Viruses

Viruses exhibit different structural forms :

1. **Helical Viruses :** Rod-shaped; genetic material is coiled within a cylindrical capsid.
2. **Spherical (Icosahedral) Viruses :** Appear round due to symmetrical arrangement of capsomeres.
3. **Complex Viruses :** Possess a head-tail structure, such as bacteriophages that infect bacteria.

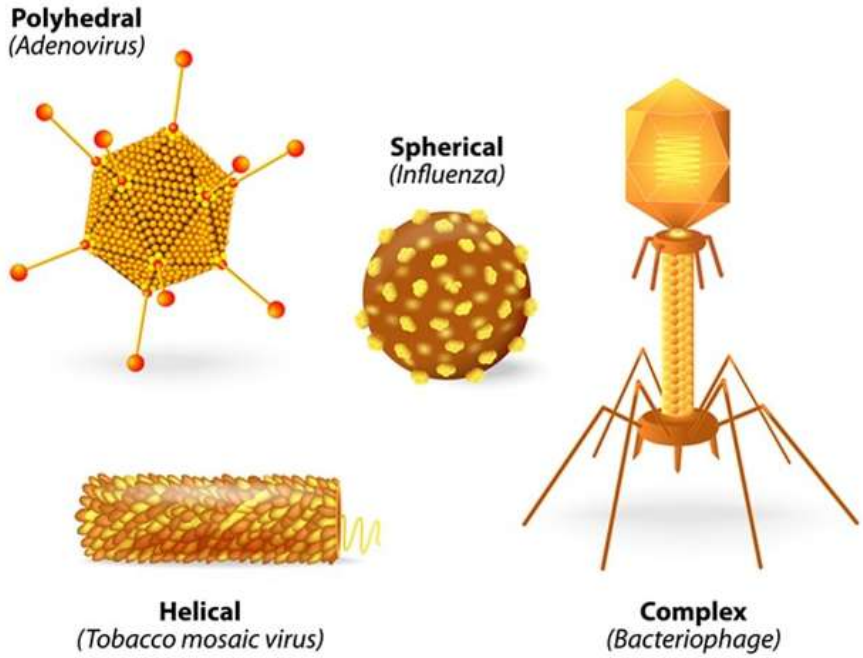


Fig. : Different Shapes of Virus

◈ Characteristics of Viruses

Viruses display both living and non-living characteristics.

Living Characteristics :

- Presence of genetic material.
- Ability to reproduce within host cells.
- Capability to mutate and evolve.

Non-Living Characteristics :

- Absence of cellular organization.
- Lack of cytoplasm and organelles.
- No independent metabolism.
- Ability to be crystallized like chemical substances.

Due to this dual nature, viruses are considered transitional forms between living and non-living entities.

Mode of Reproduction

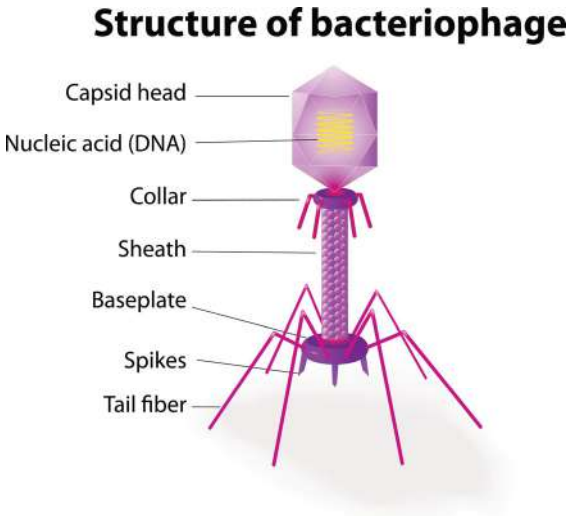
Viruses reproduce only within living cells through a process involving several stages:

1. **Attachment** – The virus binds to specific receptors on the host cell surface.
2. **Penetration** – The viral genetic material enters the host cell.
3. **Replication** – The host cell machinery synthesizes viral components.
4. **Assembly** – New viral particles are assembled.
5. **Release** – Newly formed viruses are released, often destroying the host cell.

Host Specificity

Viruses are highly specific to their hosts. A virus that infects plants generally does not infect animals. Similarly, bacteriophages infect only bacteria.

Host specificity depends on the presence of suitable receptors on host cells.



Diseases Caused by Viruses : Viruses are responsible for numerous diseases affecting plants, animals, and humans.

In Humans :

- Influenza
- Measles
- Polio
- Hepatitis
- COVID-19

In Plants :

- Tobacco mosaic disease
- Leaf curl disease

In Animals :

- Rabies
- Foot-and-mouth disease

◈ Economic and Scientific Importance

Although viruses are pathogenic in many cases, they have significant scientific importance :

- Used in vaccine development.
- Applied in gene therapy and genetic engineering.
- Useful in molecular biology research.
- Bacteriophages are employed to control harmful bacteria.

EXERCISE-1

➢ Very Short Answer Type Questions

Q.1 Name the group of plants which produce seeds but not fruits.

Q.2 In which animal phylum body cavity appears for the first time ?

Q.3 What is the basic unit of classification ?

Q.4 Name the highest taxonomic category.

Q.5 Who proposed binomial momenclature ?

Q.6 What are viruses?

Q.7 Why are viruses called obligate intracellular parasites?

Q.8 Name the two types of genetic material found in viruses.

Q.9 What is a capsid?

Q.10 Name the virus that infects bacteria.

➢ Short Answer Type Questions – Type I

Q.11 Define prokaryotes.

Q.12 Name the reproductive organs of -
(i) Gymnosperms (ii) Angiosperms.

Q.13 Which phylum includes segmented worms ?

Q.14 Name two groups of warm blooded animals with four-chambered heart.

Q.15 What are bryophytes ?

Q.16 Explain why viruses are considered at the boundary of living and non-living.

Q.17 Describe the structure of a virus.

Q.18 Write the steps involved in viral reproduction.

Q.19 Differentiate between viruses and bacteria.

Q.20 Mention four viral diseases in humans.

➢ Short Answer Type Questions – Type II

Q.21 Differentiate between dicotyledonae and monocotyledonae plants.

Q.22 Write any two important features found in platyhelminthes.

Q.23 Describe the body structure of sponges.

Q.24 Describe the 4 characteristic features of the largest phylum of kingdom Animalia.

Q.25 With the help of flow chart depict five kingdoms classification.

Q.26 How do animals of Porifera differ from animals of Cnidaria ?

Q.27 What is difference between bilateral symmetry and radial symmetry ?

Q.28 What do you mean by the term coelomate animals ? Give two examples.

Q.29 What are adaptations in fishes due to which they are aquatic ?

Q.30 Describe some flight adaptations of the birds.

➢ Long Answer Type Questions

Q.31 What is hierarchy of categories ? Describe the same.

Q.32 Describe the characteristics of kingdom Protista. Give two examples.

Q.33 Define pteridophytes. Compare between bryophytes and Pteriodophytes.

Q.34 Explain general characteristics of phylum Platyhelminthes and Nematoda. Give two examples of each.

Q.35 What are differences between the following :
(a) Cartilaginous fishes and bony fishes ;
(b) Amphibia and Reptilia ;

Q.36 Explain the structure, characteristics, and mode of reproduction of viruses.

Q.37 Describe the economic importance and harmful effects of viruses.

➢ Practical & Value Based Type Questions

Q.38 During a field-trip some students visited an agricultural farm and saw a few birds eating earthworms. They enjoyed the scene and then they also started picking and killing the earthworms for pleasure. The farmer strongly objected and asked them to leave the field.
(a) What could be the reason behind such a behavior of the farmer ?
(b) What values do you find missing in the student's behavior ?
(c) Which phylum do earthdown belong to ?
(d) Write two identifying features of earthworm.

Q.39 Rahul is having a beautiful pet dog 'Ginger'. One day, he observed a small insect between his toes. He removed it carefully from the toe and observed it curiously as chapter titled 'Diversity In Living Organisms' was being taught in the school.
(a) Identify the phylum to which the insect belong to.
(b) Enlist any two characteristic features of this phylum.
(c) Comment on Rahul's behavior.

Q.40 Ajay and mother were walking in the garden. Ajay saw mushrooms Growing on decaying leaves. He tried to uproot them. He was stopped by his mother.
(a) Identify the kingdom to which the organism belongs to.
(b) Write the mode of nutrition exhibited by the mushrooms.
(c) Why did Ajay's mother stopped him ?

Q.41 Rohit was plying cricket in the ground with his friends. One of his friend who was standing near the boundary felt something climbing upon his feet. Subsequently he started crying in pain. Rohit being class nine students identified the climbing creature as leech and helped removing the leech.
(a) Name the phylum to which leech belongs.
(b) Write two identifying features of the phylum.
(c) Enlist two values shown by Rohit.

Q.42 A group of children were playing a heap of sand. One of them collected some shelled creatures from the sand. Being taught lesson on diversity in School, one of them identified them as snail.
(a) Write two characteristic features of the phylum.
(b) What positive attitude towards learning is shown by the students ?

EXERCISE-2

Q.1 Algae are –
(A) Autotropic (B) Heterotropic
(C) Both (D) None

Q.2 Chlamydomonas is the example of -
(A) Algae (B) Fungi
(C) Lichens (D) None

Q.3 Which class is called amphibians of the plant kingdom -
(A) Thallophyta
(B) Bryophyta
(C) Pteridophyta
(D) Gymnosperm

Q.4 Example of pteridophyta is -
(A) Selaginella (B) Cycas
(C) Pinus (D) None

Q.5 Cnidoblasts is characteristic feature of -
(A) Coelenterata (B) Porifera
(C) Protozoa (D) Arthropoda

Q.6 Palemon (prawn) is the example of -
(A) Arthropoda (B) Annelida
(C) Coelenterata (D) Protozoa

Q.7 Four chambered heart is found in -
(A) Draco
(B) Hemidactylus
(C) Tortoises and turtles
(D) Crocodile

Q.8 In mollusca respiratory organ is called -
(A) Ctenidia (B) Lungs
(C) Book-lungs (D) All the above

Q.9 In larval stage which symmetry is present in echinodermata -
(A) Radial (B) Biradial
(C) Bilateral (D) None of these

Q.10 Whittaker failed to give any place to one of the following in his classification -
(A) Cyanobacteria (B) Virus
(C) Slime moulds (D) all above

Q.11 Parazoa includes -
(A) Protozoans
(B) Porifera
(C) Parasites of invertebrates (mosozoa)
(D) None above

Q.12 Viruses are essentially made up of -
(A) Proteins and nucleic acid
(B) Proteins and carbohydrates
(C) Lipids and nucleic acids
(D) Starch, proteins and lipid

Q.13 Prokaryotes are included in the group -
(A) Monera (B) Basidiomycetes
(C) Bryophytes (D) Tracheophyta

Q.14 Four kingdom system of classification was given by -
(A) Copeland (B) Whittaker
(C) Linnoeus (D) Von mohl

Q.15 Which group of plants are called "Reptiles of plant world" -
(A) Pteridophytes (B) Gymnosperms
(C) Algae (D) Fungi

Q.16 Which of the following organism has characters of both animals and plants -
(A) Blue-green algae (B) Euglena
(C) Moss (D) Cycas

Q.17 Vascular bundle are found in -
(A) Thallophyta (B) Bryophyta
(C) Pteridophyta (D) Lichens

Q.18 A branch of biology which deals with the identification, nomenclature and classification of organisms is called -
(A) Morphology (B) Ecology
(C) Taxonomy (D) Phytogeography

Q.19 Binomial nomenclature was introduced by -
(A) John Ray
(B) A. P. de Candolle
(C) A. L. de Jussion
(D) Carolus Linnaeus

Q.20 Plants with hidden reproductive organs are included in -
(A) Gymnosperms (B) Angiosperms
(C) Phanerogans (D) Cryptogams

Q.21 The basic unit of classification is -
(A) Family (B) Genus
(C) Species (D) Order

Q.22 Which of the following kingdoms includes many kinds of unicellular eukaryotic organisms ?
(A) Monera (B) Protista
(C) Fungi (D) Animalia

Q.23 Whale belongs to the class -
(A) Pisces (B) Amphibia
(C) Reptilia (D) Mammalia

Q.24 Which group of animals are called 'flat worms'?
(A) Porifera (B) Coelenterata
(C) Platyhelminthes (D) Nematoda

Q.25 Sweat and oil glands are present in the skin of -
(A) Amphibians (B) Reptiles
(C) Aves (D) Mammals

Q.26 You are identifying a plant that possesses seeds but not fruits. It may belong to -
(A) Pteridophyta (B) Gymnosperm
(C) Bryophyta (D) Angiosperm

Q.27 Seeds are naked in -
(A) Gymnosperm (B) Angiosperm
(C) Both (A) and (B) (D) Monocots

Q.28 Two chambered heart occurs in -
(A) crocodiles (B) fish
(C) aves (D) amphibians

Q.29 Skeleton is made entirely of cartilage in -
(A) Sharks (B) Tuna
(C) Rohu (D) None of these

Q.30 One of the following is not an Annelid -
(A) Nereis (B) Earthworm
(C) Leech (D) Urchins

Q.31 Viruses are called acellular because they:
(A) Are very small
(B) Lack cellular organization
(C) Cause diseases
(D) Contain DNA

Q.32 A virus contains :
(A) Both DNA and RNA
(B) Only DNA
(C) Only RNA
(D) Either DNA or RNA

Q.33 The protein coat of a virus is called :
(A) Capsule (B) Capsid
(C) Cytoplasm (D) Envelope

Q.34 Viruses reproduce :
(A) By binary fission
(B) By budding
(C) Only inside host cells
(D) Independently

Q.35 The virus that infects bacteria is called:
(A) Retrovirus (B) Bacteriophage
(C) Retroplasm (D) Pathogen

Q.36 Who first discovered viruses?
(A) Robert Hooke
(B) Louis Pasteur
(C) Dmitri Ivanovsky
(D) Darwin

Q.37 Viruses can be seen only with:
(A) Simple microscope
(B) Compound microscope
(C) Electron microscope
(D) Naked eye

Q.38 Which of the following is a viral disease?
(A) Malaria (B) Tuberculosis
(C) Measles (D) Typhoid

Q.39 The outer lipid layer present in some viruses is called :
(A) Capsomere (B) Envelope
(C) Core (D) Cytoplasm

Q.40 Viruses are considered non-living outside the host because they :
(A) Are too small
(B) Cannot move
(C) Show no metabolic activity
(D) Lack DNA

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

Q.1 Planaria is kept in which group –
(A) Coelentrata (B) Platyhelminthes
(C) Nematoda (D) Annelida

Q.2 Which of the following is a example of Bryophyte -
(A) Moss (B) Fern
(C) Pinus (D) Algae

Q.3 Dissimilarity found in Aves and Mammalia is -
(A) Warm Blooded Animal
(B) Lay eggs
(C) Breathe through Lungs
(D) Four chambered Heart

Q.4 Which one of the following plant belong to division Pteridophyta ?
(A) Funaria (B) Cycas
(C) Riccia (D) Fern

Q.5 Locomotion by tube feet is found in phylum
(A) Mollusca (B) Coelenterata
(C) Echinodermata (D) Annelida

Q.6 Example of coprophagy is
(A) horse (B) man
(C) cow (D) rabbit

Q.7 Which group of animals are found only in sea ?
(A) Mollusca (B) Echinodermata
(C) Porifera (D) Protozoa

Q.8 Match the name of phylym given under column-I, with the name of animal given under column-II and state the correct sequence alphabetically.

Column I	**Column II**
(i) Protozoa	(a) Sponge
(ii) Porifera	(b) Apis
(iii) Coelenterata	(c) Amoeba
(iv) Arthropoda	(d) Hydra

	(i)	**(ii)**	**(iii)**	**(iv)**
(A)	a	b	c	d
(B)	d	c	b	a
(C)	c	a	d	b
(D)	b	d	a	c

Q.9 If a particular animal has shelled egg, hair and teats on the body and has cloaca, then it may be a connecting link between -
(A) Reptile and aves
(B) Aves and mammal
(C) Reptile and mammal
(D) Mammal and aves

Q.10 In national parks, conservation is provided to
(A) only plant species
(B) only animal species
(C) both plants and animals
(D) complete ecosystem

Q.11 Botanical name of amla is :
(A) Medicago sativa
(B) Emblica officinalis
(C) Zinblica officinate
(D) Ocimum sanctum

Q.12 Binomial nomenclature was introduced by :
(A) John Ray
(B) Aristotle
(C) A.P De Candolle
(D) Carolus Linnaeus

Q.13 Which one of the following class of animals has coelomic cavity filled with blood ?
(A) Nematoda (B) Annelida
(C) Arthropoda (D) Protozoa

Q.14 Which two of the following animals belong to the same Phylum ?
(A) Starfish and Nereis
(B) Antedon and Starfish
(C) Antedon and Chiton
(D) Nereis and Chiton

Q.15 Which one of the following is having conducting tissue ?
(A) Fern (B) Funaria
(C) Riccia (D) Marchantia

Q.16 Which of the following two are only applicable for birds ?
(A) Warm blooded and four chambered heart
(B) Feather and breathing by lungs
(C) Warm blooded and egg laying
(D) Egg laying and with Feather

Q.17 Which of the following statement is true about dicotyledons.
(A) Their root system consists of similar fibrous roots
(B) Their leaves have paralled venation
(C) Their vascular bundles are arranged in a ring
(D) Their flowers are trimerous

Q.18 A plant has body differentiated into root, stem & leaves. It also has tissue for conduction of water and other substances, but has naked embryo. Under which type will such plants be classified ?
(A) Gymnosperms (B)Angiosperms
(C) Bryophyta (D) Pteridophyta

Q.19 Choose the correct hierarchy in classification group.
(A) Phylum - class - order - family - species
(B) Phylum - order - class - family - species
(C) Phylum - class - family - order - species
(D) Phylum - order - family - class - species

Q.20 All the cats in the world belong to the same species.
(A) Panthera leo (B) Panthera tigris
(C) Panthera pardus (D) Phelis domestics

Q.21 ..... this animal lays eggs as reptiles but has mammary glands and hair on the skin like mammals.
(A) Duckbill (B) Peripatus
(C) Bear (D) Lungfish

Q.22 A fragment of DNA that provides complete information about one protein is referred to as for that protein.
(A) Nucleotide (B) Enzyme
(C) Nucleoside (D) Gene

Q.23 The endosperm of angiosperms is
(A) haploid (B) diploid
(C) triploid (D) polyploid

Q.24 Match the following

Column I	Column II
(1) collar cells	(a) coelenterate
(2) diploblast	(b) porifera
(3) closed circulatory system	(c) echinodermata
(4) water vascular system	(d) Annelida

(A) 1-a, 2-b, 3-c, 4-d (B) 1-a, 2-b, 3-c, 4-d
(C) 1-b, 2-a, 3-d, 4-c (D) 1-a, 2-b, 3-d, 4-c

Q.25 Botanical name of Tulsi is
(A) Ocimum sanctum
(B) Saraca indica
(C) Ficus benghalensis
(D) Eagle marmelos

Q.26 The group of amphibian plants is
(A) Funaria, Marchantia
(B) Marsilia, Horse-tail
(C) Pinus, Cycas
(D) Typha, Hydrilla.

Q.27 The example of an egg laying mammal is
(A) Bat (B) Whale
(C) Echidna (D) Kangaroo

Q.28 Green gland is present in
(A) the body of pigeon
(B) the body of prawn
(C) the body of rohu fish
(D) in cactus plant

Q.29 Botanical name of Amla is.....
(A) Ocimum sanctum
(B) Phyllanthus emblica
(C) Saraca indica
(D) Ficus bengalensis

Q.30 Which plant leaves have a parallel venation ?
(A) Peepal leaves
(B) Hibiscus leaves
(C) Banana leaves
(D) Banyan tree leaves

Q.31 Obelia belongs to –
(A) Coelenterata (B) Porifera
(C) Annelida (D) Arthropoda

Q.32 In binomial nomenclature first word indicates –
(A) Species (B) Genus
(C) Sub species (D) None of these

Q.33 Whale belongs to -
(A) Mammalia (B) Amphibia
(C) Annelida (D) Reptilia

Q.34 Botanical name of Margosa (Neem) is:
(A) Azadirachta indica
(B) Pisum sativum
(C) Cassia fistula
(D) Brassica compestris

Q.35 Which is the odd one ?
(A) Planaria (B) Liver-fluke
(C) Ascaris (D) Tape-worm

Q.36 An egg laying mammal is
(A) Kangaroo (B) Bat
(C) Whale (D) Echidna

Q.37 Animals of which phylum are pseudocoelomate ?
(A) Porifera (B) Platyhelminthes
(C) Aschelminthes (D) Mollusca

Q.38 The example of hydrophytes are
(A) Hydrilla, Calotropis
(B) Lotus, Salsola
(C) Moss, Lichen
(D) Segetaria, Trapa

Q.39 Water vascular system is found in
(A) Cnidaria
(B) Echinodermata
(C) Mollusca
(D) Annelida

Q.40 Amphibians of plant kingdoms are
(A) Bacteria (B) Gymnosperm
(C) Bryophyta (D) Algae

Q.41 From the following which animal is warm blooded, presence of mammary glands and body divided into head, neck, trunk and tail.
(A) Penguin (B) Tortoise
(C) Pigeon (D) Bat

Q.42 Body structure of different animals is given below. Identify to which phylum the animal belongs.
(1) Long, cylindrical, metamerically segmented.
(2) Triploblastic, bileterally symmetrical, eucoelomate.
(3) They have setae or parapodia or suckers for locomotion.
(A) Arathropoda (B) Annelida
(C) Aschelminthes (D) Mollusca

ANSWER KEY

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	A	B	A	A	A	D	A	B	B	A	A	A	A	A
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	C	D	D	C	B	D	C	D	B	A	B	A	D
Ques.	31	32	33	34	35	36	37	38	39	40					
Ans.	B	D	B	C	B	C	C	C	B	C					

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	B	D	C	D	B	C	C	D	B	D	C	B	A
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	A	A	D	A	D	C	C	A	A	C	A	B	C
Ques.	31	32	33	34	35	36	37	38	39	40	41	42			
Ans.	A	B	A	A	C	D	C	D	B	C	D	B			

SOLUTIONS

EXERCISE-1

➢ Very Short Answer Type Questions

Sol.1 Gymnosperm produce seeds but not frutis.

Sol.2 In Annelida body cavity appeared for the first time.

Sol.3 Species is the basic unit of classification.

Sol.4 Kingdom is the highest taxonomic category.

Sol.5 Carolus Linnaeus proposed binomial nomenclature.

Sol.6 Viruses are acellular, microscopic infectious agents that contain genetic material enclosed within a protein coat and can reproduce only inside a living host cell.

Sol.7 Because they can reproduce only inside the living cells of a host organism.

Sol.8 DNA or RNA (only one type in a single virus).

Sol.9 The protective protein covering that surrounds the genetic material of a virus.

Sol.10 Bacteriophage.

➢ Short Answer Type Questions – Type I

Sol.11 Prokaryotes are microscopic organisms that have neither a distinct nucleus nor other specialized organelles.

Sol.12 Reproductive organs in gymnosperms are male and female cones.
In angiosperms, flowers are the reproductive organs.

Sol.13 Annnelida includes segmented worms.

Sol.14 Aves and mammalia are two groups of warm blooded animals with four-chambered heart.

Sol.15 Bryophytes are simplest, primitive and non-vascular plant. They are also known as amphibian of plant kingdom.

Sol.16 Viruses show both living and non-living characteristics.
Living characteristics include presence of genetic material, ability to reproduce (inside host), and mutation. Non-living characteristics include absence of cellular structure, lack of metabolism, and ability to be crystallized. Therefore, they are placed at the borderline of living and non-living organisms.

Sol.17 A virus consists of genetic material (DNA or RNA) enclosed within a protein coat called capsid. The capsid is made of protein subunits called capsomeres. Some viruses also possess an outer lipid envelope derived from the host cell membrane. The structure is simple and lacks cytoplasm or organelles.

Sol.18 The steps of viral reproduction are:
1. Attachment to host cell.
2. Entry of genetic material.
3. Replication using host cell machinery.
4. Assembly of new viral particles.
5. Release of new viruses, often destroying the host cell.

Sol.19

Feature	Viruses	Bacteria
Cellular structure	Acellular	Cellular
Size	Smaller	Larger
Genetic material	DNA or RNA (one type)	Both DNA and RNA
Reproduction	Only inside host	Independent reproduction
Living nature	Borderline	Living organism

Sol.20
1. Influenza
2. Measles
3. Polio
4. COVID-19

➢ Short Answer Type Questions – Type II

Sol.21

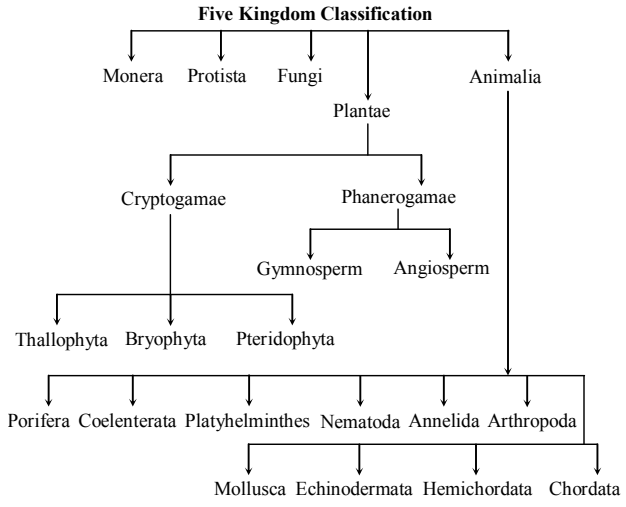
Dicotyledonae	Monocotyledonae
(i) The seeds have two cotyledons	(i) They have only one cotyledon
(ii) Leaves have reticulate venation	(ii) Leaves show parallel venation
(iii) They have tap root system	(iii) They have fibrous root system

Sol.22 Two important features found in platyhelminthes are :
(i) body is dorsoventrally flattened.
(ii) body is soft and unsegmented.

Sol.23 Sponges are simplest, multicellular organism with cellular level of organization. They are mostly asymmetrical and body is perforated by numerous pores called ostia. Body contains a cavity called as spongocoel.

Sol.24 Largest phylum of kingdom animalia is Arthropoda. The characteristic features are :
(i) Triploblastic organism with bilaterally symmetry and segmentation.
(ii) Each body segment usually bears paired lateral and jointed legs or appendages
(iii) Coelom is reduced and body is covered with a thick chitinous covering
(iv) Respiration through general body surface by gills, air tubes or book-lungs.

Sol.25



Sol.26

Porifera	Cnidaria
(i) They contain numerous pores or holes in the body.	(i) They contain a body cavity, which consist of a single opening
(ii) They possess an exoskeleton	(ii) They do not possess an exoskeleton
(iii) They are mostly asymmetrical	(iii) They are radially symmetrical
(iv) They have cellular level of organisation	(iv) They have tissue level of organisation

Sol.27

Bilateral	Radial
The animal's body can be divided into two equal halves by one plane Eg-Earthworm and cockroach	The animal's body can be divided into two equal halves by any vertical plane passing through the central axis Eg-Hydra and starfish

Sol.28 Animals that possess a true body cavity called coelom lined by mesoderm are termed as coelomate animals.
Eg. : Annelida, Arthropoda etc.

Sol.29 Adaptations in fishes for aquatic life are :
(i) Body is stream-lined to cut water current
(ii) Respiratory organ are gills
(iii) They have muscular tail and fins for movement.

Sol.30 Flight adaptations in birds :
(1) Bones have air cavities. This makes the bird light
(2) Body is covered with soft feathers
(3) The forelimbs are modified to form wings
(4) Lungs have additional bag like membranous extensions called as air sacs

➢ Long Answer Type Questions

Sol.31 Hierarchy means ranking a living creature according to their growing similarities.
The hierarchial order of classifying organism is :
Kingdom → Phylum/ Division → Class → Order → Family → Genus → Species.

(i) Kingdom : It is the highest category.
(ii) Phylum/ Division : Many classes with some common characters
(iii) Class : Similar orders are place together in a class
(iv) Order : A number of families having several common characters
(v) Family : Group of Genera having several common characters
(vi) Genus : It is a group of closely related species with common ancestory
(vii) Species : Group of organisms capable of interbreeding to produce offspring

Sol.32 Characteristics of kingdom protista are :
(i) They are unicelluar eukaryotes
(ii) Most of them are aquatic
(iii) They are autotroph as well as heterotroph
Eg. : Amoeba, Euglena and Slime moulds.

Sol.33 Pteridophyta is a vascular plant which do not have flower and donot produce seeds.

Pteridophytes	Bryophytes
1. Vasular tissues are present 2. Body is differentiated into leaf, stem and root. Eg-Fern	1. Vasular tissue are absent 2. Plant body is differentiated into leaf, stem and root like structures Eg -Moss

Sol.34 Platyhelminthes :
(i) They are dorsoventrally flat hence called as "Flat-worm"
(ii) They exhibit bilateral symmetry
(iii) They are acoelomate
(iv) They are triploblastic animals with organs level of organization
(v) Suckers and hooks are usually present
Eg. : Liver flukes, Tapeworm, Planaria

Nematoda :
(i) They are cylindrical in shape
(ii) They are parasitic or free-living with complete alimentary canal
(iii) They are triploblastic, unsegmented and show bilateral symmetry
(iv) Body cavity is not a true coelom, but a pseudocoelom
(v) Sexes are separate
Eg- Ascaris ,Wuchereria

Sol.35

(a) Cartilaginous	Bony fishes
1. Endoskeleton is made up of cartilage 2. Mouth ventral 3. Gills are naked 4. No swim bladder present Eg : Scoliodon, Trygon, Torpedo	1. Endoskeleton is entirely made up of bone 2. Mouth terminal 3. Gills are covered by operculum 4. Swim bladderis usually present Eg : Labeorohita, Anabas

(b) Amphibia	Reptilia
1. They live on land as well as in water 2. Skin is smooth and moist 3. Fertilization is external	1. They live usually on land 2. Skin is dry, hard and scaly 3. Fertilization is internal

Sol.36 Viruses are acellular infectious particles composed of genetic material enclosed within a protein coat called capsid. The genetic material may be DNA or RNA. Some viruses possess an outer lipid envelope. They lack cytoplasm, organelles, and cellular machinery.
Viruses show both living and non-living characteristics. They contain genetic material and can reproduce inside host cells, but they lack metabolism and cellular organization.
The reproduction process includes attachment to host cell, penetration, replication using host machinery, assembly of viral components, and release of new viruses. This process often damages or destroys the host cell.
Thus, viruses are unique biological entities that depend entirely on host cells for survival.

Sol.37 Viruses cause many diseases in plants, animals, and humans, leading to economic loss. In humans, they cause influenza, measles, hepatitis, and COVID-19. In plants, they cause tobacco mosaic disease

and leaf curl disease, reducing agricultural productivity. In animals, they cause rabies and foot-and-mouth disease.

Despite their harmful effects, viruses are useful in biotechnology and medicine. They are used in vaccine production, gene therapy, and molecular biology research. Bacteriophages help control harmful bacteria.

Therefore, viruses have both harmful and beneficial impacts on society.

➢ Practical & Value Based Type Questions

Sol.38 (a) The possible reason could be farmer knows the value of earthworm that how they help in making soil fertile for agriculture

(b) Students showed mercyless behaviour towards earthworm and do not know the value of every species present in nature

(c) Earthworm belongs to annelida

(d) Two identifying feature of earthworm are :

(i) These are elongated with metamerically segmented body

(ii) They have true coelom

Sol.39 (a) Insect belongs to phylum Arthropoda

(b) Two characteristic features of this phylum are

(i) Jointed appendages

(ii) Coelom is reduced and body is covered with a thick chitinous covering

(c) Rahul showed a caring behaviour towards his dog and also a behavior of curious student who wants to learn from nature

Sol.40 (a) Organism belongs to Kingdom fungi

(b) Mushrooms are saprotrophic because they fed on dead and decaying matter

(c) Ajay's mother stopped him because mushroom may have been poisonous and some toxic substances may have caused irritation in Rahul's hands

Sol.41 (a) Leech belongs to Annelida

(b) Two characteristic featuresof the phylum are

(i) They have true body cavity

(ii) They have metamerically segmented body

(c) Rohit showed caring nature for his friend and also he showed that he is a studious student who studies his chapter well.

Sol.42 (a) Two features of phylum mollusca are:

(i) They have soft and unsegmented body

(ii) Body is divided into head, visceral mass and ventral foot

(b) They study well and have better understanding of chapter

EXERCISE-2

Sol.1 **[A]**

Algae are able to produce their own food.

Sol.2 **[A]**

Chlamydomonas is the example of algae.

Sol.3 **[B]**

Bryophyta is called amphibians of plant kingdom.

Sol.4 **[A]**

Selaginella is an example of pteridophyta

Sol.5 **[A]**

Coelenterates possess special cells bearing stinging capsules called cnidoblast.

Sol.6 **[A]**

Prawn belongs to Arthropoda.

Sol.7 **[D]**

Crocodile being reptile possess 4-chambered heart.

Sol.8 **[A]**

Respiration in mollusca is by ctenidia.

Sol.9 [C]
ln larval stage bilateral symmetry is present in echinodermata and radially symmetrical in adult.

Sol.10 [B]
Whittaker failed to give any place to virus in his classification.

Sol.11 [B]
Parazoa includes porifera, where cells are loosely aggregated.

Sol.12 [A]
Viruses are essentially made up of proteins and nucleic acid.

Sol.13 [A]
Monera consist of unicellular prokaryotes.

Sol.14 [A]
Four kingdom system of classification was given by copeland.

Sol.15 [A]
Pteridophytes are called reptiles of plant world.

Sol.16 [B]
Euglena shows both autotrophic and heterotrophic nutrition.

Sol.17 [C]
Vascular bundles are found in Pteridophytes.

Sol.18 [C]
A branch of biology which deals with the identification, nomenclature and classification of organisms is called taxonomy.

Sol.19 [D]
Binomial nomenclature was introduced by carolus linnaeus.

Sol.20 [D]
Crytogams have hidden reproductive organs.

Sol.21 [C]
The basic unit of classification is species.

Sol.22 [B]
Protista contain unicellular eukaryotes.

Sol.23 [D]
Whale belongs to the class mammalia.

Sol.24 [C]
Platyhelminthes are also called flatworms.

Sol.25 [D]
Sweat and oil glands are present in the skin of mammals.

Sol.26 [B]
Gymnosperm are group of plans producing naked seeds..

Sol.27 [A]
Seeds are naked in gymnosperm.

Sol.28 [B]
Two chambered heart occurs in fish.

Sol.29 [A]
Sharks are chondriochythes.

Sol.30 [D]
Urchins comes under phylum Echinodermata.

Sol.31 [B]
Viruses do not have cellular structure such as cytoplasm or organelles.

Sol.32 [D]
A virus contains only one type of nucleic acid—either DNA or RNA.

Sol.33 [B]
Capsid is the protein covering protecting viral genetic material.

Sol.34 [C]
Viruses depend on host cell machinery for reproduction.

Sol.35 [B]
Bacteriophages specifically infect bacteria.

Sol.36 [C]
Ivanovsky first observed virus-like agents in tobacco mosaic disease.

Sol.37 [C]
Viruses are too small to be seen under a light microscope.

Sol.38 [C]
Measles is caused by a virus; others are caused by bacteria or protozoa.

Sol.39 [B]
Some viruses possess a lipid envelope derived from host cell membrane.

Sol.40 [C]
Outside host cells, viruses remain inactive and show no metabolism.

EXERCISE-3

Sol.1 [B]
Planaria are flat-worm which comes under the group of platyhelminthes.

Sol.2 [A]
Moss is an example of bryophyte.

Sol.3 [B]
Mammals are viviparous and aves are oviparous.

Sol.4 [D]
Fern belong to division pteridophyta.

Sol.5 [C]
Tube feet is found in phylum Echinodermata.

Sol.6 [D]
They eat their own faeces.

Sol.7 [B]
Echinodermata are exclusively marine.

Sol.8 [C]
Protozoa–Amoeba, Porifera–Sponge, Coelenterata–Hydra, Arthropoda–Apis.

Sol.9 [C]
These character shows connecting link between reptiles and mammal.

Sol.10 [D]
In national parks, conservation is provided to complete ecosystem.

Sol.11 [B]
Botanical name of amla is emblica officinalis. Medicago sativa–Alfalfa, Ocimum sanctum–Tulsi, Zingiber officinale–Ginger.

Sol.12 [D]
Binomial nomenclature was introduced by carolus linnaeus.

Sol.13 [C]
Arthropoda has coelomic cavity filled with blood.

Sol.14 [B]
They belong to phylum echinodermata.

Sol.15 [A]
Fern belongs to pteridophyta.

Sol.16 [B]
Presence of feather and breathing by lungs having airsacs are present only in birds.

Sol.17 [C]
In dicots vascular bundles are arranged in a ring.

Sol.18 [A]
Gymnosperms have naked seeds.

Sol.19 [A]
The hierarchy in classification is kingdom. Phylum - Class - Order - Family - Genus - species.

Sol.20 [D]
All cats in the world belong to Felis domesticus.

Sol.21 [A]
Duckbill shows characters of both reptiles and mammals.

Sol.22 [D]
A fragment of DNA that provides complete information about one protein is referred to as genes.

Sol.23 [C]
Endosperm is formed due to triple fusion.

Sol.24 [C]
Collar cells - porifera, diploblast - coelenterate, closed circulatory system - annelida, water vascular system - echinodermata.

Sol.25 [A]
Botanical name of Tulsi is Ocimum sanctum. Saraca indica-Ashoka, Ficus benghalensis-Banyan tree, Aegle marmelos-Indian bael.

Sol.26 [A]
They are examples of bryophytes.

Sol.27 [C]
The example of an egg laying mammals is echidna.

Sol.28 [A]
Preen gland are oil glands present in birds.

Sol.29 [B]
The botanical name of Amla is Phyllanthus emblica, Ocimum sanctum-Tulsi, Saraca indica-Ashoka, Ficus bengalensis-Banyan tree.

Sol.30 [C]
Banana are monocots having parallel venation.

Sol.31 [A]
Obelia belongs to coelenterata.

Sol.32 [B]
In binomial nomenclature first word indicates Genus and second word indicate Species.

Sol.33 [A]
Whale belongs to mammalia.

Sol.34 [A]
Botanical name of Margosa is Azadirachta indica. Pisum sativum-Pea, Cassia fistula-laburnum, Brassica compestris-Fields mustard.

Sol.35 [C]
Ascaris belongs to Nematoda and rest all belong to Platyhelminthes.

Sol.36 [D]
Echidna is an egg laying mammal.

Sol.37 [C]
Animals of aschelminthes are pseudocoelomate.

Sol.38 [D]
Segetaria and trapa are the examples of hydrophytes.

Sol.39 [B]
Echinodermata has water vascular system.

Sol.40 [C]
Amphibians of plant kingdom are bryophyta.

Sol.41 [D]
Bat is a mammal.

Sol.42 [B]
Annelida are metamerically segmented, triploblastic, bilaterally symmetrical, coelomate. They have parapodia or setae or suckers for locomotion.

MATHEMATICS

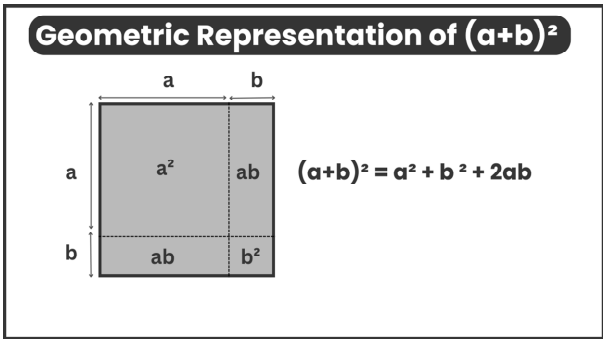
Chapter

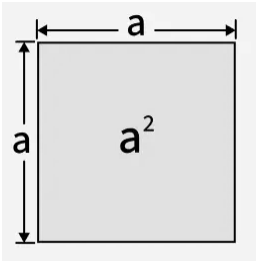
4

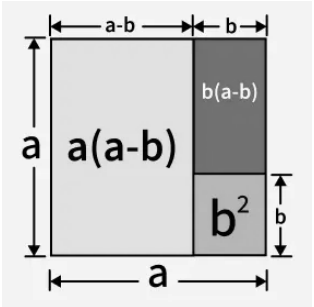
EXPLORING ALGEBRAIC IDENTITIES

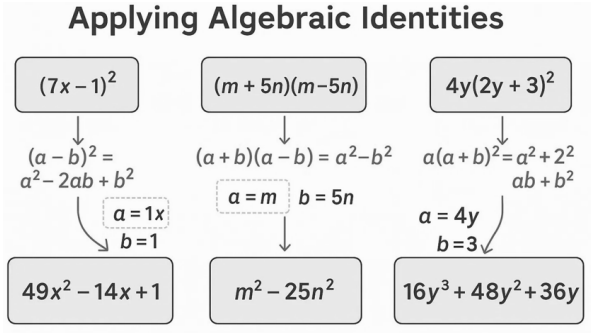
Chapter Outline

- Algebraic Identities
- Visualising Algebraic identities Using Geometric Models
- Factorization
- Rational Expressions
- H.C.F. and L.C.M. of Polynomials

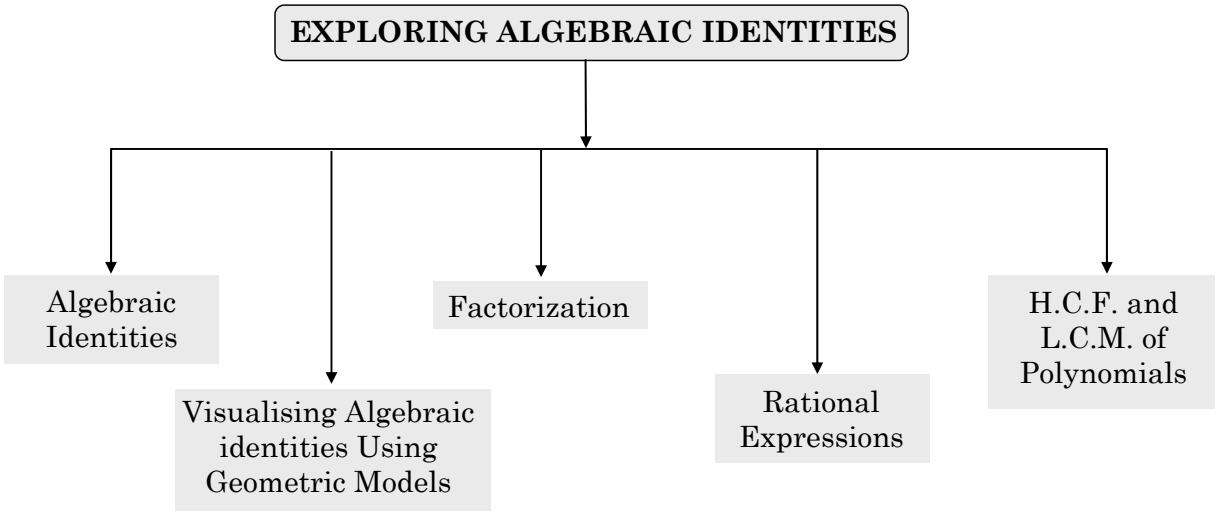








MIND MAP


EXPLORING ALGEBRAIC IDENTITIES
Algebraic Identities
Visualising Algebraic identities Using Geometric Models
Factorization
Rational Expressions
H.C.F. and L.C.M. of Polynomials

EXPLORING ALGEBRAIC IDENTITIES

Algebraic Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a – b)^2 = a^2 – 2ab + b^2$

(iii) $a^2 – b^2 = (a + b)(a – b)$

(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(v) $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

(vi) $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

(vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(viii) $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$

(ix) $a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ac)$

Special case : if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

Visualising Algebraic identities Using Geometric Models

Algebraic identities are equations that are always true for all values of variables. In the previous classes, we have read about the following identities

$(a + b)^2 = a^2 + 2ab + b^2$, $(a – b)^2$, $= a^2 – 2ab + b^2$ and $a^2 – b^2 = (a + b)(a – b)$.

In this section, we shall revise previous identities and learn some new ones. We will also visualize identities geometrically and deal with various problems based on these identities.

Identity 1 : $(a + b)^2 = a^2 + 2ab + b^2$

Proof : LHS $= (a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b)$

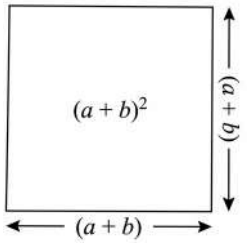
$= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2 =$ RHS.

Geometric Model

Algebraic identity $(a + b)^2$ can be derived in algebraic form by the geometrical approach.

Take a square whose side length is $(a + b)$ units.

Therefore, the area of the square is $(a + b)^2$ sq. units.

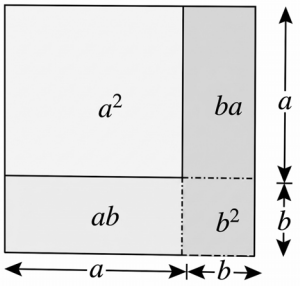


Now divide this square into four different parts vertically and horizontally such that the lengths of the parts are a units and b units.

Draw lines inside the square so that it forms :

- One larger square of side a units.
- One small square of side b units.
- Two rectangles of dimensions $a \times b$ and $b \times a$.

The area of the square $(a + b)^2$ is also equal to the sum of the areas of the individual squares and rectangles.



So, sum of areas of parts = $a^2 + ba + ab + b^2$

Therefore, $(a + b)^2 = a^2 + ba + ab + b^2 = a^2 + 2ab + b^2$

Identity 2 : $(a – b)^2 = a^2 – 2ab + b^2$

Proof : LHS = $(a – b)^2 = (a – b)(a – b) = a(a – b) – b(a – b)$

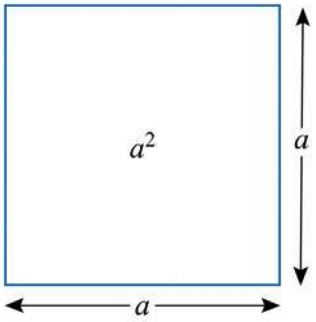
$= a^2 – ab – ab + b^2 = a^2 – 2ab + b^2$ = RHS.

Geometric Model

Let us consider a square whose sidelength is a units, such that its area is equal to a^2 sq. units

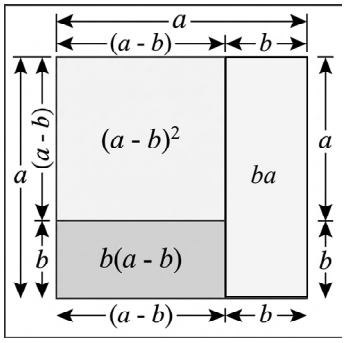
For proving $(a – b)^2$ geometrically, we have to split the square into three different geometrical shapes.

First, take a small square from this, whose sidelength is (a - b) as shown in the following figure.



Here, we get two rectangles: one is horizontal and the other is vertical

Let's add the areas of all these three figures.



Area of square with side $(a – b) = (a – b)^2$

Area of vertical rectangle = $a \times b = ab$

Area of horizontal rectangle = $(a – b) \times b = b(a – b)$

Area of square with side a = Area of square with side $(a – b)$ + Area of two rectangles

$\Rightarrow a^2 = (a – b)^2 + ab + b(a – b)$

$\Rightarrow a^2 = (a – b)^2 + ab + ab – b^2$

$\Rightarrow a^2 = (a – b)^2 + 2ab – b^2$

Therefore, $(a – b)^2 = a^2 – 2ab + b^2$

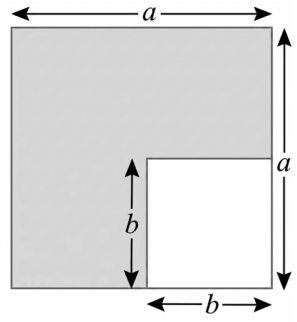
Identity 3 : $a^2 - b^2 = (a + b)(a - b)$

Proof : RHS = $(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2 = a^2 - b^2 =$ LHS .

Geometric Model

Take a square whose sidelength is a units. Therefore, the area of the square is a^2 sq. units.



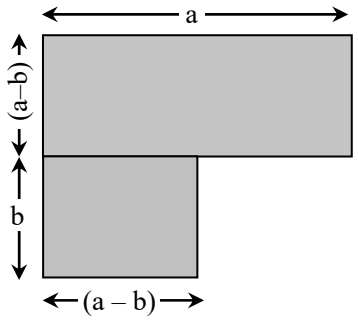
Draw a small square with the sidelength of b units at a corner of the square.

So, the area of the small square is b^2 Therefore, area of remaining shape (new geometric shape) = $a^2 - b^2$

Now split the shape into two different rectangles. The width of the larger rectangle is (a – b), and that of the small rectangle is b.

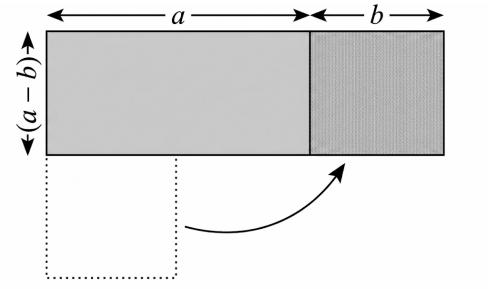
Now, cut out the small rectangle and put it next to the larger rectangle as shown.

$\therefore$ The area of the combined rectangle = $(a + b)(a - b)$



This rectangle has the same area as the original shape.

$\therefore a^2 - b^2 = (a + b)(a - b)$



Identity 4: $(x + a)(x + b) = x^2 + (a + b)x + ab$

Proof : LHS = $(x + a)(x + b) = x(x + b) + a(x + b)$

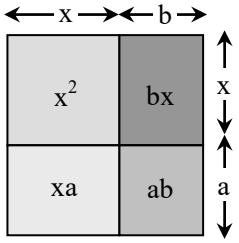
$= x^2 + bx + ax + ab = x^2 + (a + b)x + ab =$ RHS.

Geometric Model

Take a rectangle. Divide it into two parts horizontally such the their lengths become x units and a units.

Now, divide the same rectangle vertically such that the length become x units and b units as shown.

The length of the side of the square is x units.
So, area of the square = x^2 units.
The length and width of rectangle are b units and x units respectively.
Then its area = bx sq. units.
The length and width of another rectangle are x units and a un respectively.
Then its area = ax sq. units.
The length and width of another rectangle are b units and a un Area of this rectangle = ba sq. units.



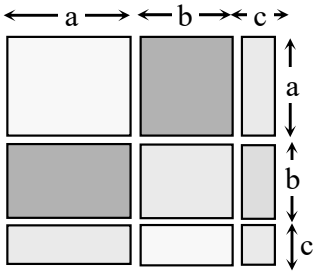
So, $(x + a)(x + b) = x^2 + xa + bx + ba$
$= x^2 + (a + b)x + ab$

Identity 5: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

Proof : LHS = $(a + b + c)^2 = (a + b + c)(a + b + c)$
$= a(a + b + c) + b(a + b + c) + c(a + b + c) = a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$
$= a^2 + b^2 + c^2 + ab + ab + ac + ac + bc + bc = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$ = RHS

Geometric Model

Take a square and divide it vertically into three different parts by drawing two lines. The lengths are a units, b units and c units respectively as shown in the figure.



Divide the square horizontally into three parts such that their sidelengths become a units, b units and c units respectively, as shown in the figure.
Now the sidelength of the original square is (a + b + c) units.
Therefore, the area of the square is
$(a + b + c) \times (a + b + c) = (a + b + c)^2$
Here, the square whose area is $(a + b + c)^2$, is divided into three square and six rectangle.
Length of each side of three squares are a units, b units and c units.
So, their areas are a^2 sq. units, b^2 sq. units and c^2 sq. units respectively.
The lengths of sides of two rectangles are a units and b units.
So, the area of each rectangle is ab sq. units.

The lengths of sides of two rectangles are c units and a units.
So, the area of each rectangle is ca sq. units.
The lengths of sides of two rectangle are b units and c units.
So, the area of each rectangle is bc sq. units.
So, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

a^2	ab	ca
ab	b^2	bc
ca	bc	c^2

Identity 6: $(a + b)^3 = a^3 + b^3 + 3ab\ (a + b) = a^3 + b^3 + 3a^2b + 3ab^2$

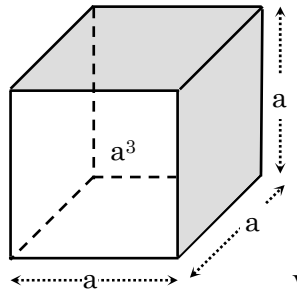
Proof : LHS $= (a + b)^3 = (a + b)\ (a + b)\ (a + b) = (a + b)\ (a + b)^2$

$= a(a^2 + b^2 + 2ab) + b(a^2 + b^2 + 2ab) = a^3 + ab^2 + 2a^2b + a^2b + b^3 + 2ab^2$

$= a^3 + b^3 + 3a^2b + 3ab^2 + a^3 + b^3 + 3ab(a + b) = RHS.$

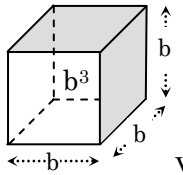
Geometric Model

Take a cube of side a units and a cube of side b units. Then take three cuboids each of whose side measure a, a and b units. Now take three cuboids each of whose sides measure b, b and a units.



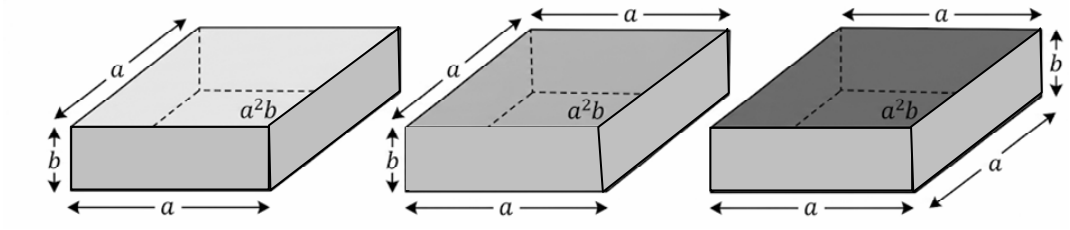
Volume = a^3

Cube of side a



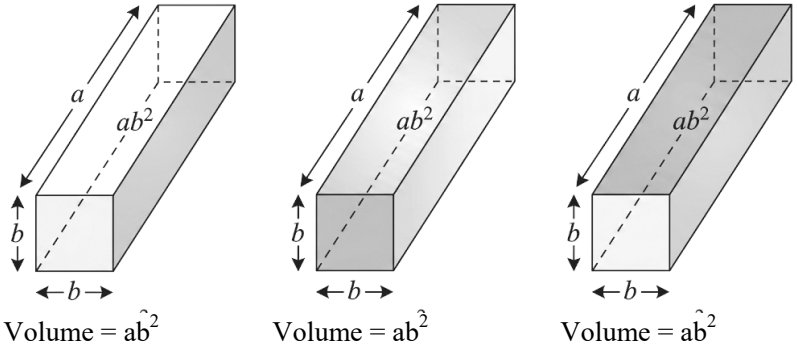
Volume = b^3

Cube of side b



Volume = a^2b Volume = a^2b Volume = a^2b

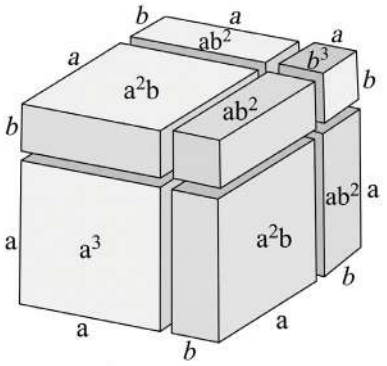
Cuboids of sides a, a, b



Volume = ab^2 Volume = ab^2 Volume = ab^2

Cuboids of sides a, b, b

Finally, places all the cubes and cuboids on a flat surface such that they form a cube of side (a + b) as shown below.



Cube of side (a + b)

So, $(a + b)^3 = a^3 + b^3 + (a^2b + a^2b + a^2b) + (ab^2 + ab^2 + ab^2)$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$= a^3 + b^3 + 3ab\,(a + b)$

Ex.1 Expand each of the following :

(i) $(3x - 4y)^2$ (ii) $\left(\frac{x}{2} + \frac{y}{3}\right)^2$

Sol. (i) We have,

$(3x - 4y)^2 = (3x)^2 - 2 \times 3x \times 4y + (4y)^2$

$= 9x^2 - 24xy + 16y^2$

(ii) We have,

$\left(\frac{x}{2} + \frac{y}{3}\right)^2 = \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{y}{3} + \left(\frac{y}{3}\right)^2$

$= \frac{x^2}{4} + \frac{1}{3}xy + \frac{y^2}{9}$

Ex.2 Find the products :

(i) $(2x + 3y)\,(2x - 3y)$ (ii) $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

Sol. (i) We have,

$(2x + 3y)\,(2x - 3y)$

$= (2x)^2 - (3y)^2$ [Using: $(a + b)(a - b) = a^2 - b^2$]

$= (2x)^2 - (3y)^2 = 4x^2 - 9y^2$

(ii) We have,

$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= \left\{(x^2)^2 - \left(\frac{1}{x^2}\right)^2\right\}\left(x^4 + \frac{1}{x^4}\right) = \left(x^4 - \frac{1}{x^4}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2 = x^8 - \frac{1}{x^8}$

Ex.3 Evaluate each of the following by using identities :

(i) 103×97 (ii) 103×103

(iii) $(97)^2$ (iv) $185 \times 185 - 115 \times 115$

Sol. (i) We have,

$103 \times 97 = (100 + 3)(100 - 3)$

$= (100)^2 - (3)^2 = 10000 - 9 = 9991$

(ii) We have,

$103 \times 103 = (103)^2$

$= (100 + 3)^2 = (100)^2 + 2 \times 100 \times 3 + (3)^2$

$= 10000 + 600 + 9 = 10609$

(iii) We have,

$(97)^2 = (100 - 3)^2$

$= (100)^2 - 2 \times 100 \times 3 + (3)^2$

$= 10000 - 600 + 9 = 9409$

(iv) We have,

$185 \times 185 - 115 \times 115$

$= (185)^2 - (115)^2 = (185 + 115)(185 - 115)$

$= 300 \times 70 = 21000$

Ex.4 If $x^2 + \frac{1}{x^2} = 34$, find : $x^4 + \frac{1}{x^4}$

Sol. We have,

$$x^2 + \frac{1}{x^2} = 34 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 1156$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 1156$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 1156 - 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 1154$$

Ex.5 If $x + y = 12$ and $xy = 32$, find the value of $x^2 + y^2$.

Sol. We have,

$(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 144 = x^2 + y^2 + 2 \times 32$ [Putting $x + y = 12$ and $xy = 32$]

$\Rightarrow 144 = x^2 + y^2 + 64$

$\Rightarrow 144 - 64 = x^2 + y^2$

$\Rightarrow x^2 + y^2 = 80$

Ex.6 Prove that : $2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = [(a - b)^2 + (b - c)^2 + (c - a)^2]$

Sol. We have,

L.H.S. $= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$

$= (a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)$ [Re-arranging the terms]

$= (a - b)^2 + (b - c)^2 + (c - a)^2$ = R.H.S.

Hence, $2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$

$= [(a - b)^2 + (b - c)^2 + (c - a)^2]$

Ex.7 Write the following in expanded form :

(i) $(9x + 2y + z)^2$ (ii) $(3x + 2y - z)^2$

(iii) $(x - 2y - 3z)^2$ (iv) $(-x + 2y + z)^2$

Sol. Using the identity

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) We have, $(9x + 2y + z)^2$

$= (9x)^2 + (2y)^2 + z^2 + 2 \times 9x \times 2y + 2 \times 2y \times z + 2 \times 9x \times z$

$= 81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18xz$

(ii) We have, $(3x + 2y - z)^2$

$= [3x + 2y + (-z)]^2$

$= (3x)^2 + (2y)^2 + (-z)^2 + 2 \times 3x \times 2y + 2 \times 2y \times (- z) + 2 \times 3x \times (-z)$

$= 9x^2 + 4y^2 + z^2 + 12xy - 4yz - 6xz$

(iii) We have, $(x - 2y - 3z)^2$

$= [x + (-2y) + (-3z)]^2$

$= x^2 + (-2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

(iv) We have, $(-x + 2y + z)^2$

$= [(-x) + 2y + z]^2$

$= (-x)^2 + (2y)^2 + z^2 + 2 \times (-x) \times (2y) + 2 \times 2y \times z + 2 \times (-x) \times z$

$= x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$

Ex.8 If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find $a + b + c$.

Sol. We know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (a + b + c)^2 = 250 + 2 \times 3$

$\Rightarrow (a + b + c)^2 = 256$

$\Rightarrow (a + b + c) = \pm \sqrt{256}$

$\Rightarrow a + b + c = \pm 16$

Ex.9 Write each of the following in expanded form:

(i) $(2x + 3y)^3$ (ii) $(3x - 2y)^3$

Sol. (i) Replacing a by 2x and b by 3y in the identity

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$, we have

$(2x + 3y)^3 = (2x)^3 + (3y)^3 + 3 \times 2x \times 3y \times (2x + 3y)$

$= 8x^3 + 27^3 + 18xy \times 2x + 18xy \times 3y$

$= 8x^3 + 27y^3 + 36x^2y + 54xy^2$

(ii) Replacing a by 3x and b by 2y in the identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$, we have

$(3x - 2y)^3 = (3x)^3 - (2y)^3 - 3 \times 3x \times 2y \times (3x - 2y)$

$= 27x^3 - 8y^3 - 18xy \times (3x - 2y)$

$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$

Ex.10 If $x + y = 12$ and $xy = 27$, find the value of$x^3 + y^3$.

Sol. We know that

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Putting $x + y = 12$ and $xy = 27$ in the above identity, we get

$12^3 = x^3 + y^3 + 3 \times 27 \times 12$

$\Rightarrow \quad 1728 = x^3 + y^3 + 972$

$\Rightarrow \quad x^3 + y^3 = 1728 - 972$

$\Rightarrow \quad x^3 + y^3 = 756$

Ex.11 If $x + y + z = 1$, $xy + yz + zx = -1$ and $xyz = -1$, find the value of $x^3 + y^3 + z^3$.

Sol. We know that :

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \Rightarrow \quad x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - 3xy - 3yz - 3zx)$

[Adding and subtracting $2xy + 2yz + 2zx$]

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = (x + y + z)\{(x + y + z)^2 - 3(xy + yz + zx)\}$

$\Rightarrow x^3 + y^3 + z^3 - 3 \times -1 = 1 \times \{(1)^2 - 3 \times -1\}$

[Putting the values of $x + y + z$, $xy + yz + zx$, and xyz]

$\Rightarrow \quad x^3 + y^3 + z^3 + 3 = 4$

$\Rightarrow \quad x^3 + y^3 + z^3 = 4 - 3$

$\Rightarrow \quad x^3 + y^3 + z^3 = 1$

Factorization

Type I : Factorization by taking out the common factors.

Ex.12 Factorize the following expression : $2x^2y + 6xy^2 + 10x^2y^2$

Sol. $2x^2y + 6xy^2 + 10x^2y^2 = 2xy(x + 3y + 5xy)$

Type II : Factorization by grouping the terms.

Ex.13 Factorize the following expression : $a^2 - b + ab - a$

Sol. $a^2 - b + ab - a$

$= a^2 + ab - b - a = (a^2 + ab) - (b + a)$

$= a(a + b) - (a + b) = (a + b)(a - 1)$

Type III : Factorization by making a perfect square.

Ex.14 Factorize the following expression : $9x^2 + 12xy + 4y^2$

Sol. $9x^2 + 12xy + 4y^2$

$= (3x)^2 + 2 \times (3x) \times (2y) + (2y)^2 = (3x + 2y)^2$

Ex.15 Factorize the following expression : $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$, $x \neq 0, y \neq 0$

Sol. $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$

$= \left(\frac{x}{y}\right)^2 + 2\left(\frac{x}{y}\right).\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = \left(\frac{x}{y} + \frac{y}{x}\right)^2$

Ex.16 Factorize the following expression : $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$, $x \neq 0$

Sol. $= \left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$

$= \left(5x - \frac{1}{x}\right)^2 + 2 \times \left(5x - \frac{1}{x}\right) \times 2 + 2^2$

$= \left(5x - \frac{1}{x} + 2\right)^2$

Type IV : Factorizing by difference of two squares.

Ex.17 Factorize each of the following expressions :

(i) $36x^2 - 12x + 1 - 25y^2$ (ii) $a^2 - \frac{9}{a^2}$, $a \neq 0$

Sol. (i) $36x^2 - 12x + 1 - 25y^2$

$= (6x)^2 - 2 \times 6x \times 1 + 1^2 - (5y)^2 = (6x-1)^2 - (5y)^2$

$= \{(6x - 1) - 5y\}\ \{(6x - 1) + 5y\}$

$= (6x - 1 - 5y)\ (6x - 1 + 5y)$

$= (6x - 5y - 1)\ (6x + 5y - 1)$

(ii) $a^2 - \frac{9}{a^2} = (a)^2 - \left(\frac{3}{a}\right)^2$

$= \left(a - \frac{3}{a}\right)\left(a + \frac{3}{a}\right)$

Ex.18 Factorize the following algebraic expression : $x^4 - 81y^4$

Sol. $x^4 - 81y^4 = [(x)^2]^2 - (9y^2)^2$

$= (x^2 - 9y^2)\ (x^2 + 9y^2)$

$= \{x^2 - (3y)^2\}\ (x^2 + 9y^2)$

$= (x - 3y)\ (x + 3y)\ (x^2 + 9y^2)$

Ex.19 Factorize the following expression: $x(x+z) - y\ (y+z)$

Sol. $x(x+z) - y\ (y+z) = (x^2 - y^2) + (xz - yz)$

$= (x-y)\ (x+y) + z\ (x-y)$

$= (x-y)\ \{(x+y) + z\}$

$= (x-y)\ (x + y + z)$

Ex.20 Factorize the following expression : $x^4 + x^2 + 1$

Sol. $x^4 + x^2 + 1 = (x^4 + 2x^2 + 1) - x^2$

$= (x^2 + 1)^2 - x^2 = (x^2 + 1 - x)(x^2 + 1 + x)$

$= (x^2 - x + 1)(x^2 + x + 1)$

Type V : Factorizing the sum and difference of cubes of two quantities.

(i) $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

(ii) $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

Ex.21 Factorize the following expression : $a^3 + 27$

Sol. $a^3 + 27 = a^3 + 3^3 = (a + 3)(a^2 - 3a + 9)$

Ex.22 Simplify : $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

Sol. Let $x + y = a$ and $x - y = b$.

Then, $ab = (x+y)(x-y) = x^2 - y^2$ and $a - b = (x+y) - (x-y) = 2y$

$\therefore (x+y)^3 - (x-y)^3 - 6y(x^2 - y^2)$

$= a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$= \{(x+y) - (x-y)\}^3 = (2y)^3 = 8y^3$

◈ Factorization of quadratic polynomial by splitting the middle team

Type I : Factorization of Quadratic polynomials of the form $x^2 + bx + c$.

Ex.23 Factorize the following quadratic polynomials : $x^2 - 21x + 108$

Sol. In order to factorize $x^2 - 21x + 108$, we have to find two numbers such that their sum is -21 and the product 108.

Clearly, $-21 = -12 - 9$ and $-12 \times -9 = 108$

$\therefore x^2 - 21x + 108 = x^2 - 12x - 9x + 108$

$= (x^2 - 12x) - (9x - 108)$

$= x(x - 12) - 9(x - 12) = (x - 12)(x - 9)$

Ex.24 Factorize the following by splitting the middle term : $x^2 + 3\sqrt{3}x + 6$

Sol. In order to factorize $x^2 + 3\sqrt{3}x + 6$, we have to find two numbers p and q such that $p + q = 3\sqrt{3}$ and $pq = 6$

Clearly, $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ and $2\sqrt{3} \times \sqrt{3} = 6$

So, we write the middle term $3\sqrt{3}x$ as $2\sqrt{3}x + \sqrt{3}x$, so that

$x^2 + 3\sqrt{3}x + 6$

$= x^2 + 2\sqrt{3}x + \sqrt{3}x + 6$

$= (x^2 + 2\sqrt{3}x) + (\sqrt{3}x + 6)$

$= (x^2 + 2\sqrt{3}x) + (\sqrt{3}x + 2\sqrt{3} \times \sqrt{3})$

$= x(x + 2\sqrt{3}) + \sqrt{3}(x + 2\sqrt{3})$

$= (x + 2\sqrt{3})(x + \sqrt{3})$

Type II : Factorization of polynomials reducible to the form $x^2 + bx + c$.

Ex.25 Factorize : $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$.

Sol. Let $a^2 - 2a = x$. Then,

$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120 = x^2 - 23x + 120$

Now, $x^2 - 23x + 120 = x^2 - 15x - 8x + 120$

$= (x^2 - 15x) - (8x - 120)$

$= x(x - 15) - 8(x - 15)$

$= (x - 15)(x - 8)$

Replacing x by $a^2 - 2a$ on both sides, we get

$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

$= (a^2 - 2a - 15)(a^2 - 2a - 8)$

$= (a^2 - 5a + 3a - 15)(a^2 - 4a + 2a - 8)$

$= \{(a(a - 5) + 3(a - 5)\}\{a(a - 4) + 2(a - 4)\}$

$= \{(a - 5)(a + 3)\}\{(a - 4)(a + 2)\}$

$= (a - 5)(a + 3)(a - 4)(a + 2)$

Ex.26 Factorize the expressions by splitting the middle term : $9(x - 2y)^2 - 4(x - 2y) - 13$

Sol. The given expression is $9(x - 2y)^2 - 4(x - 2y) - 13$.

Putting $x - 2y = a$, we get

$9(x - 2y)^2 - 4(x - 2y) - 13 = 9a^2 - 4a - 13$

Now, $9a^2 - 4a - 13 = 9a^2 - 13a + 9a - 13$

$= (9a^2 - 13a) + (9a - 13)$

$= a(9a - 13) + (9a - 13)$

$= (a + 1)(9a - 13)$

Replacing a by $x - 2y$ on both sides, we get

$9(x - 2y)^2 - 4(x - 2y) - 13 = (x - 2y + 1)\{9(x - 2y) - 13\}$

$= (x - 2y + 1)(9x - 18y - 13)$

Ex.27 If $x^2 + px + q = (x + a)(x + b)$, then factorize $x^2 + pxy + qy^2$.

Sol. We have,

$x^2 + px + q = (x + a)(x + b)$

$\Rightarrow \quad x^2 + px + q = x^2 + x(a + b) + ab$

On equating the coefficients of like powers of x, we get

$p = a + b$ and $q = ab$

$\therefore \quad x^2 + pxy + qy^2 = x^2 + (a + b)xy + aby^2$

$= (x^2 + axy) + (bxy + aby^2)$

$= x(x + ay) + by(x + ay)$

$= (x + ay)(x + by)$

Type III : Factorization of quadratic polynomials of the form $ax^2 + bx + c$, $a \neq 0, 1$

Ex.28 Factorize the following expression : $6x^2 - 5x - 6$

Sol. The given expression is of the form $ax^2 + bx + c$, where $a = 6$, $b = -5$ and $c = -6$.

In order to factorize the given expression, we have to find two numbers ℓ and m such that

$\ell + m = b = -5$ and $\ell m = ac = 6 \times -6 = -36$

i.e., we have to find two factors of -36 such that their sum is -5. Clearly,

$-9 + 4 = -5$ and $-9 \times 4 = -36$

$\therefore \quad \ell = -9$ and $m = 4$

Now, we split the middle term $-5x$ of $6x^2 - 5x - 6$ as $-9x + 4x$, so that

$6x^2 - 5x - 6 = 6x^2 - 9x + 4x - 6$

$= (6x^2 - 9x) + (4x - 6)$

$= 3x(2x - 3) + 2(2x - 3) = (2x - 3)(3x + 2)$

Ex.29 Factorize : $\sqrt{3}\,x^2 + 11x + 6\sqrt{3}$

Sol. The given quadratic expression is of the form $ax^2 + bx + c$, where $a = \sqrt{3}$, $b = 11$ and $c = 6\sqrt{3}$.

In order to factorize it, we have to find two numbers ℓ and m such that

$\ell + m = b = 11$ and $\ell m = ac = \sqrt{3} \times 6\sqrt{3} = 18$

Clearly, $9 + 2 = 11$ and $9 \times 2 = 18$

$\therefore \quad \ell = 9$ and $m = 2$

Now, $\sqrt{3}\,x^2 + 11x + 6\sqrt{3}$

$= \sqrt{3}\,x^2 + 9x + 2x + 6\sqrt{3}$

$= (\sqrt{3}\,x^2 + 9x) + (2x + 6\sqrt{3})$

$= (\sqrt{3}\,x^2 + 3\sqrt{3} \times \sqrt{3}\,x) + (2x + 6\sqrt{3})$

$= \sqrt{3}\,x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3})$

$= (\sqrt{3}\,x + 2)(x + 3\sqrt{3})$.

Hence, $\sqrt{3}\,x^2 + 11x + 6\sqrt{3}$

$= (\sqrt{3}\,x + 2)(x + 3\sqrt{3})$

Ex.30 Factorize the following by splitting the middle term : $\frac{1}{3}x^2 - 2x - 9$

Sol. In order to factorize $\frac{1}{3}x^2 - 2x - 9$, we have to find to number ℓ and m such that

$\ell + m = -2$ and $\ell m = \frac{1}{3} \times -9 = -3$

Clearly, $-3 + 1 = -2$ and $-3 \times 1 = -3$

So, we write the middle term $-2x$ as $-3x + x$, so that

$\frac{1}{3}x^2 - 2x - 9 = \frac{1}{3}x^2 - 3x + x - 9$

$= (\frac{1}{3}x^2 - 3x) + (x - 9) = (\frac{1}{3}x^2 - \frac{9}{3}x) + (x - 9)$

$= \frac{1}{3}x(x - 9) + (x - 9) = (x - 9)\left(\frac{1}{3}x + 1\right)$

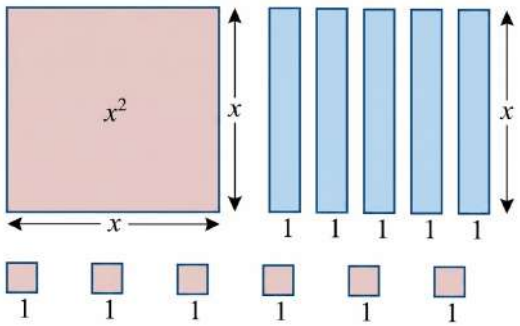
◈ Visualising Factorisation of quadratic expressions through algebra tiles

A quadratic expression like $x^2 + bx + c$ can be interpreted as the area of a rectangle, which can be decomposed into smaller rectangles and squares.

- We break the expression into parts (areas).
- By rearranging these shapes, we can determine the **length** and **breadth**, which represent the factors.

Thus, factorisation means **finding dimensions of a rectangle whose area equals the given algebraic expression.**

Consider the expression: $x^2 + 5x + 6$



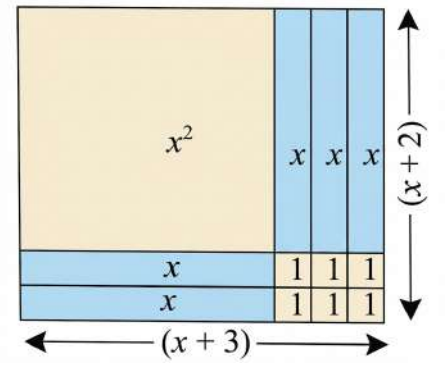
Step 1 : We interpret each term geometrically.

Term	Geometric Meaning
x^2	Area of square of side x
5x	Area of rectangle with side x and 5
6	Area of rectangle 2 × 3 or constant area

Thus, the total area represents a large rectangle formed by combining these shapes.

Step 2 : Represent each term as area

- Square of area = x^2 sq. units
- Rectangle of area = 5x sq. units.
- Rectangle of area = 6 sq. units



Step 3 : Split the middle term

We split (5x) so that arranging rectangles becomes easy. We need two numbers such that :

Sum = 5 and product = 6. These numbers are 2 and 3.

So, $5x = 2x + 3x$

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

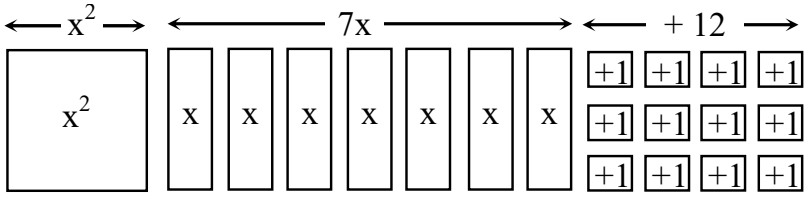
$= (x^2 + 2x) + (3x + 6) = x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$

Ex.31 Solving $x^2 + 7x + 12$ using algebra tiles.

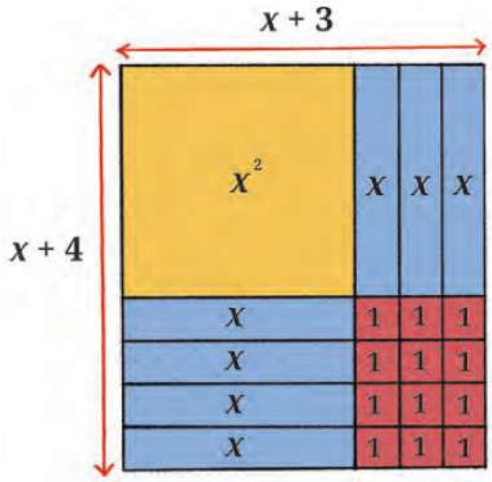
Sol. Given

$x^2 + 7x + 12$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$x^2 + 7x + 12$

$= (x + 3)(x + 4)$

◈ Factorization of $x^3 + y^3 + z^3 - 3xyz$

- In order to factorize the algebraic expressions of the form $x^3 + y^3 + z^3 - 3xyz$

  We use the following identity :

  $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

- If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Ex.32 Factorize : $(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$

Sol. We have,

$(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$

$= \{(a+b) + (b+c) + (c+a)\}\{(a+b)^2 + (b+c)^2 + (c+a)^2 - (a+b)(b+c) - (b+c)(c+a) - (c+a)(a+b)\}$

$= (2a+2b+2c)$

$\{(a^2 + 2ab + b^2) + (b^2 + 2bc + c^2) + (c^2 + 2ca + a^2) - (ab + ac + b^2 + bc) - (bc + ba + c^2 + ca) - (ca + cb + a^2 + ab)\}$

$= 2(a+b+c)(2a^2 + 2b^2 + 2c^2 + 2ab + 2bc + 2ca - ab - ac - b^2 - bc - bc - ba - c^2 - ca - ca - cb - a^2 - ab)$

$= 2(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Ex.33 Resolve $a^3 - b^3 + 1 + 3ab$ into factors.

Sol. $a^3 - b^3 + 1 + 3ab$

$= a^3 + (-b)^3 + 1^3 - 3(a)(-b)(1)$

$= (a - b + 1)(a^2 + b^2 + 1 + ab - a + b)$

$= (a - b + 1)(a^2 + b^2 + ab - a + b + 1)$

Ex.34 Factorize : $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$.

Sol. $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$

$= (\sqrt{2}\,a)^3 + (2b)^3 - (3c)^3 - 3(\sqrt{2}\,a)(2b)(-3c)$

$= (\sqrt{2}\,a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ac)$

Ex.35 Prove that : $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$

Sol. We have,

$a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$= \frac{1}{2}(a + b + c)\{(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)\}$

$= \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$

◈ Factorization of algebraic expression of the form $a^3 + b^3 + c^3$, when $a + b + c = 0$

Ex.36 Simplify : $\dfrac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$

Sol. We have,

$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$= 3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

Similarly,

$(a - b) + (b - c) + (c - a) = 0$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

$\therefore \dfrac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$

$= \dfrac{3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)}{3(a - b)(b - c)(c - a)}$

$= (a + b)(b + c)(c + a)$

Ex.37 Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.

Sol. We have, $x^3 - 8y^3 - 36xy - 216$

$= x^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6)$

$= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$

$= 0 \times (x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$

$[\because x = 2y + 6 \Rightarrow x - 2y - 6 = 0] = 0$

Simplifying Rational Expressions

A rational expression is an algebraic fraction in which both the numerator and denominator are polynomials and the denominator is not zero.

Thus, $\frac{p(x)}{q(x)}$, where p(x) and q(x) are polynomials and $q(x) \neq 0$ is a rational expression.

For example : $\frac{x+2}{x}, \frac{x^2+2}{x+3}$ and $\frac{3x}{2x+5}$ are rational expressions.

Note: Every polynomial p(x) is a rational expression, since we can write p(x) as $\frac{p(x)}{1}$.

Simplifying Rational Expressions

Simplifying a rational expression means **reducing it to its simplest form** by canceling common factors from the numerator and denominator.

Important Rule :

You can cancel **common factors only**, not terms. $\frac{(x+2)(x-3)}{(x+2)} = x - 3$

Steps to Simplify Rational Expressions

(i) Factorise numerator and denominator completely.

(ii) Identify common factors in numerator and denominator.

(iii) Cancel common factors.

(iv) Write the simplified expression.

Ex.38 Simplify ; $\frac{6x}{3}$

Sol. Factorise ; $\frac{6x}{3} = \frac{3 \times 2x}{3} = 2x$ [Cancel common factor 3, we get, 2x]

Ex.39 Simplify : $\frac{x^2 - 9}{x - 3}$

Sol. Factorising numerator, we get, $x^2 - 9 = (x - 3)(x + 3)$

Now, $\frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{x-3} = (x + 3)$ [Cancel common factor $(x - 3)$, we get $(x + 3)$]

Ex.40 Simplify : $\frac{2x^2 + 4x}{2x}$

Sol. Factorising numerator, $\frac{2x(x+2)}{2x} = (x + 2)$ [Cancel common factor 2x, we get $(x + 2)$]

◈ Application of Factor Theorem in the Factorization of Polynomials

(i) Obtain the polynomial p(x)

(ii) Obtain the constant term in p(x) and find its all possible factors. For example, in the polynomial $x^4 + x^3 - 7x^2 - x + 6$ the constant term is 6 and its factors are ± 1, ± 2, ± 3, ± 6.

(iii) Take one of the factors, say a and replace x by it in the given polynomial. If the polynomial reduces to zero, then (x – a) is a factor of polynomial.

(iv) Obtain the factors equal in no. to the degree of polynomial. Let these are (x–a), (x–b), (x–c.).....

(v) Write p(x) = k (x–a) (x–b) (x–c) ..... where k is constant.

(vi) Substitute any value of x other than a,b,c ..... and find the value of k.

Ex.41 Using factor theorem, factorize the polynomial $x^3 - 6x^2 + 11x - 6$.

Sol. Let $f(x) = x^3 - 6x^2 + 11x - 6$

The constant term in f(x) is equal to – 6 and factors of – 6 are ±1, ± 2, ± 3, ± 6.

Putting x = 1 in f(x), we have

$f(1) = 1^3 - 6 \times 1^2 + 11 \times 1 - 6$

$= 1 - 6 + 11 - 6 = 0$

$\therefore$ (x– 1) is a factor of f(x)

Similarly, x – 2 and x – 3 are factors of f(x).

Since f(x) is a polynomial of degree 3. So, it can not have more than three linear factors.

Let f(x) = k (x–1) (x– 2) (x – 3). Then,

$x^3 - 6x^2 + 11x - 6 = k(x-1)(x-2)(x-3)$

Putting x = 0 on both sides, we get

$-6 = k(0-1)(0-2)(0-3)$

$\Rightarrow -6 = -6k \Rightarrow k = 1$

Putting k = 1 in f(x) = k (x – 1) (x – 2) (x – 3), we get

f(x) = (x – 1) (x – 2) (x – 3)

Hence, $x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

Ex.42 Factorize, $2x^4 + x^3 - 14x^2 - 19x - 6$

Sol. Let $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ be the given polynomial. The factors of the constant term – 6 are ±1, ±2, ±3 and ±6, we have,

$f(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6$

$= 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$

and,

$f(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6 = 32 - 8 - 56 + 38 - 6 = 0$

So, x + 1 and x + 2 are factors of f(x).

$\Rightarrow$ (x + 1) (x + 2) is also a factor of f(x)

$\Rightarrow x^2 + 3x + 2$ is a factor of f(x)

Now, we divide

$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ by

$x^2 + 3x + 2$ to get the other factors.

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\
\hline
x^2 + 3x + 2 & 2x^4 + x^3 - 14x^2 - 19x - 6 \\
 & 2x^4 + 6x^3 + 4x^2 \\
 & - \quad - \quad - \\
\hline
 & -5x^3 - 18x^2 - 19x - 6 \\
 & -5x^3 - 15x^2 - 10x \\
 & + \quad + \quad + \\
\hline
 & -3x^2 - 9x - 6 \\
 & -3x^2 - 9x - 6 \\
 & + \quad + \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore \quad 2x^4 + x^3 - 14x^2 - 19x - 6$

$= (x^2 + 3x + 2)(2x^2 - 5x - 3)$

$= (x + 1)(x + 2)(2x^2 - 5x - 3)$

Now $2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$

$= 2x(x - 3) + 1(x - 3)$

$= (x - 3)(2x + 1)$

Hence, $2x^4 + x^3 - 14x^2 - 19x - 6$

$= (x + 1)(x + 2)(x - 3)(2x + 1)$

Ex.43 Factorize $x^2 + 4 + 9z^2 + 4x - 6xz - 12z$

Sol. The presence of the three squares viz. x^2, $(2)^2$, and $(3z)^2$ gives a clue that identity (vii) could be used. So we write.

$A = x^2 + (2)^2 + (3z)^2 + 4x - 6xz - 12z$

We note that the last two of the product terms are negative and that both of these contain z. Hence we write A as

$A = x^2 + (2)^2 + (-3z)^2 + 2.2x - 2.x.(-3z) + 2.2(-3z) = (x + 2 - 3z)^2$

$= (x + 2 - 3z)(x + 2 - 3z)$

COMPETITIVE LEVEL

H.C.F. and L.C.M. of Polynomials

The relation between H.C.F. and L.C.M. of two polynomials is the product of the two polynomials is equal to the product of their H.C.F. and L.C.M.

If p(x) and q(x) are two polynomials, then $p(x) \cdot q(x)$ = {H.C.F. of p(x) and q(x)} x {L.C.M. of p(x) and q(x)}.

Ex. 44 Find the H.C.F. and L.C.M. of the expressions $a^2 - 12a + 35$ and $a^2 - 8a + 7$ by factorization.

Sol. First expression $= a^2 - 12a + 35 = a^2 - 7a - 5a + 35 = a(a - 7) - 5(a - 7) = (a - 7)(a - 5)$

Second expression $= a^2 - 8a + 7 = a^2 - 7a - a + 7 = a(a - 7) - 1(a - 7) = (a - 7)(a - 1)$

Therefore, the H.C.F. = $(a - 7)$ and L.C.M. = $(a - 7)(a - 5)(a - 1)$

Note:

(i) The product of the two expressions is equal to the product of their factors.

(ii) The product of the two expressions is equal to the product of their H.C.F. and L.C.M.

Product of the two expressions $= (a^2 - 12a + 35)(a^2 - 8a + 7)$

$= (a - 7)(a - 5)(a - 7)(a - 1)$

$= (a - 7)(a - 7)(a - 5)(a - 1)$

= H.C.F. × L.C.M. of the two expressions

Ex.45 Find the L.C.M. of the two expressions $a^2 + 7a - 18$, $a^2 + 10a + 9$ with the help of their H.C.F.

Sol. First expression $= a^2 + 7a - 18 = a^2 + 9a - 2a - 18 = a(a + 9) - 2(a + 9) = (a + 9)(a - 2)$

Second expression $= a^2 + 10a + 9 = a^2 + 9a + a + 9 = a(a + 9) + 1(a + 9) = (a + 9)(a + 1)$

Therefore, the H.C.F. $= (a + 9)$

Therefore, L.C.M. = Product of the two expressions/H.C.F.

$$= \frac{(a^2 + 7a - 18)(a^2 + 10a + 9)}{(a + 9)} = \frac{(a + 9)(a - 2)(a + 9)(a + 1)}{(a + 9)} = (a - 2)(a + 9)(a + 1)$$

Ex.46 $m^2 - 5m - 14$ is an expression. Find out another similar expression such that their H.C.F. is $(m - 7)$ and L.C.M. is $m^3 - 10m^2 + 11m + 70$.

Sol. According to the problem,

$$\text{Required Expression} = \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Given expression}}$$

$$= \frac{(m^3 - 10m^2 + 11m + 70)(m - 7)}{m^2 - 5m - 14}$$

$$= \frac{(m^2 - 5m - 14)(m - 5)(m - 7)}{m^2 - 5m - 14}$$

$$m^2 - 5m - 14 \overline{\left| \begin{array}{l} m^3 - 10m^2 + 11m + 70 \\ m^3 - 5m^2 - 14m \\ \hline - \quad + \quad + \\ \hline -5m^2 + 25m + 70 \\ -5m^2 + 25m + 70 \\ \hline + \quad - \quad - \\ \hline 0 \end{array} \right.} \; m - 5$$

$= (m - 5)(m - 7) = m^2 - 12m + 35$

Therefore, the required expression $= m^2 - 12m + 35$

EXERCISE-1

➢ Very Short Answer Type Questions

Q.1 Use suitable identities to find the following products :
(i) $(x + 4)(x + 10)$
(ii) $(4m + 7n)^2$

Q.2 Evaluate the following products without multiplying directly.
(i) 95×96 (ii) 104×96

Q.3 Write the following cubes in expanded form.
(i) $(2x + 1)^3$ (ii) $\left[x - \frac{2}{3}y\right]^3$

Q.4 Without actually calculating the cubes, find the value of $(28)^3 + (-15)^3 + (-13)^3$

Q.5 Simplify : $\frac{x^2 + 5x}{x}$

Q.6 For what value of k is $y^3 + ky + 2k - 2$ exactly divisible by $(y + 1)$?

➢ Short Answer Type Questions – Type I

Q.7 Evaluate the following using suitable identities :
(i) $(99)^3$ (ii) $(102)^3$

Q.8 Simplify : $\frac{x^2 + 2x - 5}{x^2 - 25}$

Factorize each of the following expression

Q.9(a) (i) $6 - 5y - y^2$
(ii) $a^2 + 46a + 205$
(iii) $ab + ac - b^2 - bc$

(b) (i) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$
(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

(c) (i) $x^2y^2 - xy - 72$
(ii) $x^4 - 5x^2 + 4$
(iii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

(d) (i) $2x^3 - 3x^2 - 17x + 30$
(ii) $x^3 - 6x^2 + 11x - 6$
(iii) $x^3 + x^2 - 4x - 4$
(iv) $3x^3 - x^2 - 3x + 1$
(v) $x^3 - 23x^2 + 142x - 120$

Q.10 If $a + b + c = 9$ and $ab + bc + ca = 40$, find $a^2 + b^2 + c^2$.

Q.11 Find positive square root of $36x^2 + 60x + 25$.

Q.12 Simplify : $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$

➢ Short Answer Type Questions – Type II

Q.13 Find the following products :
(i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$
(ii) $(x^2 - 1)(x^4 - 1)(x^4 + x^2 + 1)$

Q.14 Factorize : $4(x-y)^2 - 12(x - y)(x + y) + 9(x+y)^2$

Q.15 Find the value of $1 - a^2 + 14ab - 49b^2$.

Q.16 Prove that :
$$\frac{0.77 \times 0.77 \times 0.77 + 0.23 \times 0.23 \times 0.23}{0.77 \times 0.77 - 0.77 \times 0.23 + 0.23 \times 0.23} = 1$$

Q.17 Prove that
$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$

Q.18 Factorize : (i) $x^6 - y^6$ (ii) $x^{12} - y^{12}$

Q.19 If $x^4 + \frac{1}{x^4} = 47$. Find the value of $x^3 + \frac{1}{x^3}$.

Q.20 If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.

Q.21 If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, prove that $a = b = c$.

Q.22 Solving $x^2 + 5x + 6$ using algebra tiles.

➢ Long Answer Type Questions

Q.23 Simplify :
$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$$

Q.24 If $a + b + c = 0$, then find the value of
$$\frac{(b + c)^2}{bc} + \frac{(c + a)^2}{ca} + \frac{(a + b)^2}{ab}$$

Q.25 Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

Q.26 Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

Q.27 If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Q.28 If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Q.29 Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Q.30 Solving $2x^2 - 7x + 3$ using algebra tiles.

EXERCISE-2

Q.1 The polynomial $11a^2 - 12\sqrt{2}a + 2$ on factorisation gives :

(A) $(11a + \sqrt{2})(a - \sqrt{2})$

(B) $(a - \sqrt{2})(11a - \sqrt{2})$

(C) $(a + 11)(a + \sqrt{2})$

(D) $(11a - \sqrt{2})(a + \sqrt{2})$

Q.2 The polynomial $x^5 - a^2x^3 - x^2y^3 + a^2y^3$ on factorisation gives :

(A) $(x - y)(x - a)(x + a)(x^2 + y^2 + xy)$

(B) $(x + a)(x - y)(x - a)(x^2 - y^2 + xy)$

(C) $(x + a)(x + y)(x - a)(x^2 + y^2 + xy)$

(D) None of these

Q.3 Factorise : $a^3 + b^3 + 3ab - 1$.

(A) $(a + b - 1)(a^2 + b^2 + a + b + 1 - ab)$

(B) $(a + b - 1)(a^2 + b^2 + a + b - 1 + ab)$

(C) $(a + b - 1)(a^2 + b^2 - a - b + 1 + ab)$

(D) None of these

Q.4 Find the square root of

$$\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}.$$

(A) $\frac{2x}{3} + \frac{3}{2x} - \frac{1}{2}$ (B) $\frac{x}{3} - \frac{3}{2x} + 1$

(C) $\frac{3}{x} + \frac{2}{3x} - \frac{1}{2}$ (D) $\frac{x}{3} + \frac{3}{2x} - \frac{1}{2}$

Q.5 If $\sqrt{4x^4 + 12x^3 + 25x^2 + 24x + 16} = ax^2 + bx + c$, then which of the following is true?

(A) $2b = a - c$ (B) $2a = b + c$

(C) $2b = a + c$ (D) $2b = c - a$

Q.6 If $a + b + c = 6$, the value of $(2 - a)^3 + (2 - b)^3 + (2 - c)^3 - 3(2 - a)(2 - b)(2 - c)$ is :

(A) 1 (B) 0 (C) –1 (D) 2

Q.7 If $\left(x + \frac{1}{x}\right) = 2$, then the value of $\left(x^6 + \frac{1}{x^6}\right)$ is -

(A) 2 (B) 4 (C) 8 (D) None

Q.8 If $(x + y + z) = 6$ and $(xy + yz + zx) = 11$, then the value of $(x^3 + y^3 + z^3 - 3xyz)$ is-

(A) 81 (B) 54

(C) 18 (D) None of these

Q.9 Find the value of

$$\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2},$$

when $a = -5$, $b = -6$, $c = 10$.

(A) 1 (B) –1 (C) 2 (D) –2

Q.10 If $a^2 - b^2 = 21$ and $a^2 + b^2 = 29$, which of the following could be the value of ab ?

I. –10 II. $5\sqrt{2}$ III. 10

(A) I only (B) II only

(C) III only (D) I and III only

Q.11 On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get :

(A) $8a^2$ (B) $8a^2b$

(C) $8a^3b$ (D) $8a^3$

Q.12 If $(x + y + z) = 1$, $xy + yz + zx = -1$, $xyz = -1$, then value of $x^3 + y^3 + z^3$ is :

(A) –1 (B) 1

(C) 2 (D) –2

Q.13 Evaluate:

$$\frac{(a - b)^2}{(b - c)(c - a)} + \frac{(b - c)^2}{(a - b)(c - a)} + \frac{(c - a)^2}{(a - b)(b - c)}$$

(A) 0 (B) 1 (C) 2 (D) 3

Q.14 The value of

$$\frac{0.76 \times 0.76 \times 0.76 + 0.24 \times 0.24 \times 0.24}{0.76 \times 0.76 - 0.76 \times 0.24 + 0.24 \times 0.24}$$ is :

(A) 0.52 (B) 1

(C) 0.01 (D) 0.1

Q.15 Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?

(A) $x^3 + y^2 + 2xy$

(B) $x^2 + y^2 - xy$

(C) xy^2

(D) $3xy$

Q.16 If $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$, then value of $x^3 - y^3$ is

(A) 1 (B) – 1

(C) 0 (D) $\frac{1}{2}$

Q.17 If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is :

(A) 0 (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q.18 If $x^2 + kx + 6 = (x + 2)(x + 3)$, for all x, then the value of k is :

(A) 1 (B) – 1

(C) 5 (D) 3

Q.19 The value of k, if x – 1 is a factor of $p(x) = kx^2 - \sqrt{2}\,x + 1$ is :

(A) $\sqrt{2}$ (B) $\sqrt{2} + 1$

(C) $\sqrt{2} - 1$ (D) $\frac{\sqrt{3}}{2}$

Q.20 If (x + 2) and (x – 1) are factors of the polynomial $10x^2 + px + q$, then $\frac{p}{q}$, is :

(A) $-\frac{1}{2}$ (B) $\frac{1}{4}$

(C) –1 (D) –2

Q.21 Which of the following is not a rational expression ?

(A) $x - \frac{1}{2}$ (B) $\frac{x^2 - \sqrt{3}}{x}$

(C) $\frac{x - 5}{x^{1/2}}$ (D) $\frac{3}{x}$

EXERCISE-3

(Previous Year Questions – NTSE, IJSO & IMO)

Q.1 If $x + \frac{1}{x} = a + b$, $x - \frac{1}{x} = a - b$ then

(A) ab = 1 (B) ab = 2

(C) a + b = 0 (D) a = b

Q.2 H.C.F. and L.C.M. of expressions $(x^3 - 1)$ and A are $(x - 1)$ and $(x^6 - 1)$ respectively. Then the value of A is:

(A) $x^3 + 1$

(B) $x^4 - x^3 + x - 1$

(C) $(x - 1)(x^2 - x + 1)$

(D) $(x - 1)(x^2 + x + 1)$

Q.3 H.C.F. of $x^2 + 5x + 6$ and $x^3 + 27$ is :

(A) x + 2 (B) x – 2

(C) x – 3 (D) x + 3

Q.4 One of the factors of the expression $(2x - 3y)^2 - 7(2x - 3y) - 30$ is :

(A) 2x – 3y – 10 (B) 2x – 3y + 10

(C) 3x – 2y + 5 (D) 6x – 4y – 15

Q.5 L.C.M. of $x^3 + x^2 + x + 1$ and $x^3 - x^2 + x - 1$ is:

(A) $x^4 + 1$ (B) $x^4 - 1$

(C) $x^2 + 1$ (D) $x^2 - 1$

Q.6 If $x + \frac{1}{x} = 3$, then the value of $x^6 + \frac{1}{x^6}$ is :

(A) 927 (B) 114 (C) 364 (D) 322

Q.7 If a, b, c and d are natural numbers such that $a^5 = b^6$, $c^3 = d^4$, and d – a = 61, then the smallest value of c – b is :

(A) 61 (B) 122 (C) 239 (D) 593

Q.8 If x, y, z are positive real numbers and a, b, c are rational numbers, then the value of

$$\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}}$$

(A) – 1 (B) 0

(C) 1 (D) None of these

Q.9 If x + 1 is a factor of $ax^3 + x^2 - 2x + 4a - 9$, then the value of a =

(A) 1 (B) 2

(C) 3 (D) 1

Q.10 If x – y = 1 and $x^2 + y^2 = 41$, then x + y =

(A) ± 6 (B) ± 9

(C) ± 16 (D) ± 25

Q.11 If $x^4 + \frac{1}{x^4} = 47$, then the value of $x + \frac{1}{x}$ =

(A) 3 (B) 5

(C) 7 (D) 11

Q.12 If a + b + c = 4 and ab + bc + ca = 6, then $a^3 + b^3 + c^3 - 3abc$ =

(A) – 6 (B) – 7

(C) – 8 (D) 10

Q.13 If $\ell + m + n = 0$, then $\frac{\ell^2}{mn} + \frac{m^2}{n\ell} + \frac{n^2}{\ell m}$ =

(A) 1 (B) 2

(C) 3 (D) 4

Q.14 If a + b + c = 5 and ab + bc + ca = 10, then $a^3 + b^3 + c^3 - 3abc$ =

(A) – 15 (B) – 20

(C) – 25 (D) 30

Q.15 If $a^2 + b^2 + c^2 = 90$ and a + b + c = 20, then the value ob ab + bc + ca =

(A) 155 (B) 160

(C) 165 (D) 170

Q.16 The rational expression $\frac{x^2 + 4x - 5}{x^2 - 1}$ in its simplest form is :

(A) x + 5 (B) $\frac{x + 5}{x - 1}$

(C) $\frac{x + 1}{x + 5}$ (D) $\frac{x + 5}{x + 1}$

ANSWER KEY

EXERCISE - 1

1. (i) $x^2 + 14x + 40$ (ii) $16m^2 + 56mn + 49n^2$ **2.** (i) 9120 (ii) 9984

3. (i) $8x^3 + 12x^2 + 6x + 1$ (ii) $x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$ **4.** 16380 **5.** (x + 5) **6.** 3

7. (i) 970299 (ii) 1061208 **8.** $\frac{(x-3)}{(x-5)}$

9. **(a)** **(i)** (1 – y) (y + 6) **(ii)** (a + 41) (a + 5) **(iii)** (a – b) (b + c)

(b) **(i)** $(\sqrt{3}x + 2)(x + 3\sqrt{3})$ **(ii)** $(\sqrt{3}x + 2)(4x - \sqrt{3})$

(c) **(i)** (xy – 9) (xy + 8) **(ii)** (x – 2) (x + 2) (x – 1) (x + 1) **(iii)** $(x - 5)(x + 1)(x - 2)^2$

(d) **(i)** (x – 2) (x + 3) (2x – 5) **(ii)** (x – 1) (x – 2) (x – 3) **(iii)** (x + 1) (x + 2) (x – 2)

(iv) (x – 1) (x + 1) (3x – 1) **(v)** (x – 1) (x – 10) (x – 12)

10. 1 **11.** (6x + 5) **12.** $(\sqrt{2}a + \sqrt{3}b)$ **13.** (i) $\frac{x^3}{8} + 8y^3$ (ii) $x^6 - 1$ **14.** $(x + 5y)^2$

15. (1 + a – 7b) (1 – a + 7b)

18. **(i)** $(x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$

(ii) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)(x^2 - xy + y^2)(x^2 + xy + y^2)$

19. ± 18 **20.** 370 **22.** (x + 3) (x + 2) **23.** (a + b) (b + c) (c + a) **24.** 3

25. $(-10y)(12x^2 + 25y^2)$ **26.** $x^3 - 8y^3 - z^3 - 6xyz$ **30.** (x – 3) (2x – 1)

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	A	D	C	B	A	C	A	D	D	B	D	B	D
Ques.	16	17	18	19	20	21									
Ans.	C	C	C	C	A	C									

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	B	D	A	B	D	D	C	B	B	A	C	C	C	A
Ques.	16														
Ans.	D														

SOLUTIONS

EXERCISE-1

➢ Very Short Answer Type Questions

1. (i) $(x + 4)(x + 10) = x^2 + (4 + 10)(x) + (4 \times 10) = x^2 + 14x + 40$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
(ii) $(4m + 7n)^2 = [(4m)^2 + 2(4m)(7n) + (7n)^2]$
$= 16m^2 + 56mn + 49n^2$
[Using $(a + b)^2 = a^2 + 2ab + b^2$]

2. (i) $95 \times 96 = (100 - 5) \times (100 - 4) = [(100) + (-5)][(100) + (-4)]$
$= (100)^2 + (-5 - 4)(100) + \{-5 \times (-4)\}$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 10000 - 900 + 20 = 9120$
(ii) $104 \times 96 = (100 + 4)(100 - 4)$
$= (100)^2 - (4)^2$
[Using $(a + b)(a - b) = a^2 - b^2$]
$= 10000 - 16 = 9984$

3. (i) $(2x + 1)^3 = (2x)^3 + 1^3 + 3(2x)^2(1) + 3(2x)(1)^2$
[Using $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$]
$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$

(ii) $\left(x - \frac{2}{3}y\right)^3 = (x^3) - \left(\frac{2}{3}y\right)^3 - 3(x)^2\left(\frac{2}{3}y\right) + 3(x)\left(\frac{2}{3}y\right)^2$
[Using $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]
$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

4. $(28)^3 + (-15)^3 + (-13)^3$
Let $a = 28$, $b = -15$ and $c = -13$. Then,
$a + b + c = 28 - 15 - 13 = 28 - 28 = 0$
$\therefore a^3 + b^3 + c^3 = 3abc$
$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 3 \times 28 \times (-15) \times (-13) = 16380.$

5. Factorise numerator : $\frac{x(x+5)}{x} = (x + 5)$
[Cancel common factor, x we get $(x + 5)$]

6. If any polynomial p(x) is exactly divisible by $(x - a)$, then remainder will be zero.
$\therefore$ $p(a) = 0$; $p(y) = y^3 + ky + 2k - 2$
Here, $a = -1$
$\therefore$ $p(-1) = 0$
$(-1)^3 + k(-1) + 2k - 2 = 0$
$-1 - k + 2k - 2 = 0$
$k - 3 = 0 \Rightarrow \boxed{k = 3}$ **Ans.**

➢ Short Answer Type Questions – Type I

7. (i) $(99)^3 = (100 - 1)^3 = (100)^3 - (1)^3 - 3(100)^2(1) + 3(100)(1)^2$
$= 1000000 - 1 - 30000 + 300$
[$(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]
$= 1000000 - 30000 + 300 - 1 = 970299$
(ii) $(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 - 3(100)^2(2) + 3(1000)(2)^2$
[Using $(a + b)^3$] $= a^3 + b^3 + 3ab^2 + 3ab^2$]
$= 1000000 + 8 + 60000 + 1200 = 1000000 + 60000 + 1200 + 8 = 1061208$

8. We have, $\frac{x^2 + 2x - 5}{x^2 - 25} = \frac{x^2 + 5x - 3x - 15}{x^2 - 5^2}$
$= \frac{(x-5)(x-3)}{(x+5)(x-5)} = \frac{(x-3)}{(x-5)}$

9. (a) (i) $6 - 5y - y^2$
$= -(y^2 + 5y - 6)$
$= -(y^2 + 6y - y - 6)$
$= -(y(y + 6) - 1(y + 6))$
$= -(y - 1)(y + 6)$ **Ans.**

(ii) $a^2 + 46a + 205$
$= a^2 + 41a + 5a + 205$
$= a(a + 41) + 5(a + 41)$
$= (a + 5)(a + 41)$ **Ans.**

(iii) $ab + ac - b^2 - bc$
$= a(a - b) + c(a - b)$
$= (a - b)(a + c)$ **Ans.**

(b) (i) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$
$= \sqrt{3}\left(x^2 + \frac{11}{\sqrt{3}}x + 6\right)$
$= \sqrt{3}\left(x^2 + \frac{4}{2\sqrt{3}}x + \frac{18}{2\sqrt{3}}x + 6\right)$
$= \sqrt{3}\left(x^2 + \frac{4}{2\sqrt{3}}\right)\left(x + \frac{9}{\sqrt{3}}\right)$

(ii) $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$

$= \sqrt{3}\left[4x^2 + \frac{5x}{\sqrt{3}} - 2\right]$

$= \sqrt{3}\left[4x^2 + \frac{5x}{\sqrt{3}} - 2\right]$

$= 4\sqrt{3}\left[x^2 + \frac{5x}{4\sqrt{3}} - \frac{2}{4}\right]$

$= 4\sqrt{3}\left[x^2 - \frac{16x}{8\sqrt{3}} - \frac{6x}{8\sqrt{3}} - \frac{1}{2}\right]$

$= 4\sqrt{3}\left[\left(x - \frac{2}{\sqrt{3}}\right).\left(x + \frac{3}{4\sqrt{3}}\right)\right]$

(c) (i) $x^2y^2 - xy - 72$

$= x^2y^2 - 9xy + 8xy - 72$

$= xy(xy - 9) + 8(xy - 9)$

$(xy + 8)(xy - 9)$

(ii) $x^4 - 5x^2 + 4$

$= x^4 - 4x^2 - x^2 + 4$

$= x^2(x^2 - 4) - 1(x^2 - 4)$

$= (x^2 - 1).(x^2 - 4)$

$= (x - 1)(x + 1)(x - 2)(x + 2)$

(iii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

$= (x^2 - 4x)^2 - (x^2 - 4x) - 20$

$= (x^2 - 4x)^2 + 4(x^2 - 4x) - 5(x^2 - 4x) - 20$

$= (x^2 - 4x)(x^2 - 4x - 5) + 4(x^2 - 4x - 5)$

$= (x^2 - 4x + 4).(x^2 - 4x - 5)$

$= (x - 2)^2.(x^2 - 5x + 4x - 5)$

$= (x - 2)^2.(x + 1)(x - 5)$

(d) (i) $2x^3 - 3x^2 - 17x + 30$

put $x = \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$

Here,

$(x - 2)$ is a factor of $p(x)$ because

$p(2) = 2.2^3 - 3.2^2 - 17.2 + 30$

$p(2) = 16 - 12 - 34 + 30$

$\boxed{p(2) = 0}$

Now,

$$\begin{array}{r|l} & 2x^2 + x - 15 \\ \hline x - 2 & 2x^3 - 3x^2 - 17x + 30 \\ & 2x^3 - 4x^2 \\ \hline & \quad x^2 - 17x + 30 \\ & \quad x^2 - 2x \\ \hline & \quad -15x + 30 \\ & \quad -15x + 30 \\ \hline & \quad 0 \end{array}$$

$\therefore 2x^3 - 3x^2 - 17x + 30$

$= (x - 2)(2x^2 - x - 15)$

$= (x - 2).(2x^2 + 6x - 5x - 15)$

$= (x - 2).(2x(x + 3) - 5(x + 3))$

$= (x - 2)(2x - 5)(x + 3)$

(ii) $x^3 - 6x^2 + 11x - 6$

$q(x) = x^3 - 6x^2 + 11x - 6$

Using Factor Theorem,

Put $x = \pm1, \pm2, \pm3,$

$\therefore p(q(1) = 13 - 6.1^2 + 11.1 - 6$

$= 1 - 6 + 11 - 6 = 0$

$q(2) = 2^3 - 6.2^2 + 11.2 - 6$

$= 8 - 24 + 22 - 6 = 0$

$q(-1) = (-1)^3 - 6(-1)^2 + 11(-1) - 6$

$= -1 - 6 - 11 - 6 = 24$

$q(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6$

$= -8 - 24 - 22 - 6 = -60$

$q(3) = 3^3 - 6.3^2 + 11.3 - 6$

$q(3) = 27 - 54 + 33 - 6 = 60 - 60 = 0$

$\therefore$ $(x - 1)$, $(x - 2)$ & $(x - 3)$ are the factors of the given polynomial.

$\therefore x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

(iii) $x^3 + x^2 - 4x - 4$

Using Factor Theorem,

Put $x = \pm1, \pm2, \pm4$

$r(1) = 1 + 1 - 4 - 4 = -6$

$r(-1) = -1 + 1 + 4 - 4 = 0$

$r(2) = 2^3 + 2^2 - 8 - 4 = 8 + 4 - 8 - 4 = 0$

$r(-2) = (-2)^3 + (-2)^2 + 8 - 4$

$= -8 + 4 + 8 - 4 = 0$

$\therefore$ $(x + 1)$, $(x - 2)$ & $(x + 2)$ are the factors of $r(x)$.

$\therefore r(x) = x^3 + x^2 - 4x - 4$

$= (x + 1)(x - 2)(x - 2)$

(iv) $f(x) = 3x^3 - x^2 - 3x + 1$

put $x = 1$,

$f(1) = 3.1^3 - 1^2 - 3.1 + 1 = 0$

$\therefore (x - 1)$ is a factor of $f(x)$

Now,

$$\begin{array}{r|l} & 3x^2 + 2x - 1 \\ \hline x - 1 & 3x^3 - x^2 - 3x + 1 \\ & 3x^3 - 3x^2 \\ \hline & \quad 2x^2 - 3x + 1 \\ & \quad 2x^2 - 2x \\ \hline & \quad -x + 1 \\ & \quad -x + 1 \\ \hline & \quad 0 \end{array}$$

$\therefore 3x^3 - x^2 - 3x + 1$

$= (x - 1).(3x^2 + 2x - 1)$

$= (x-1).(3x^2 + 3x - x - 1)$

$= (x-1).(3x(x+1) - 1(x+1))$

$= (x-1)(3x-1)(x+1)$

(v) $g(x) = x^3 - 23x^2 + 142x - 120$

put $x = \pm1, \pm2, \pm3, \pm4$ ----- ±120

$\therefore g(1) = 1 - 23 + 142 - 120$

$g(1) = 0 \Rightarrow (x-1)$ is a factor of $g(x)$

$\therefore g(x) = (x-1).k(x)$

To find k(x), use long division method,

$$\begin{array}{r|l} & 3x^2 - 22x + 120 \\ \hline x-1 & x^3 - 23x^2 + 142x - 120 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & -22x^2 + 142x - 120 \\ & -22x^2 + 22x \\ & + \quad - \\ \hline & 120x - 120 \\ & 120x - 120 \\ & - \quad + \\ \hline & 0 \end{array}$$

$\therefore g(x) = (x-1)(x^2 - 22 + 120)$

$= (x-1)(x2 - 12x - 10x + 120)$

$= (x-1)(x-10)(x-12)$ **Ans.**

10. Given that, $a + b + c = 9$ and $ab + bc + ca = 40$,

$a^2 + b^2 + c^2 = ?$

Now, using the Identity,

$(a+b+c)^2 = (a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(40)$

$\Rightarrow a^2 + b^2 + c^2 = 81 - 80 = 1$ **Ans.**

11. Square root of $36x^2 + 60x + 25$

Using Factorization,

$36x^2 + 60x + 25 = (6x)^2 + 2.(6x).5 + 5^2$

$= (6x+5)^2$

$\therefore \sqrt{36x^2 + 60 + 25} = \pm(6x + 5)$ **Ans.**

12. $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$

$= \sqrt{(\sqrt{2}a)^2 + 2.(\sqrt{2}a)(\sqrt{3}b) + (\sqrt{3}b)^2}$

$= \sqrt{(\sqrt{2}a + \sqrt{3}b)^2} = \pm(\sqrt{2}a + \sqrt{3}b)$

➢ Short Answer Type Questions – Type II

13. (i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$

$= \left[\left(\frac{x}{2}\right) + (2y)\right]\left[\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right]$

$= \left(\frac{x}{2}\right)^3 + (2y)^3$

[Using $(a+b)(a^2 - ab + b^2) = a^3 + b^3$]

$= \frac{x^3}{8} + 8y^3$

(ii) $(x^2 - 1)(x^4 + x^2 + 1)$

$= [(x^2) - (1)][(x^2)^2 + (x^2)(1) + (1)^2]$

$= (x^2)^3 - (1)^3 = x^6 - 1$

[Using $(a-b)(a^2 + ab + b^2) = a^3 - b^3$]

14. $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$

$= (2(x-y))^2 - 2.2(x-y).3(x+y) + (3(x+y))^2$

$= [2(x-y) - 3(x+y)]^2$

$= (2(x-y) - 3x - 3y)^2$

$= (-x - 5y)^2 = (x + 5y)^2$

$= (x + 5y)(x + 5y)$ **Ans.**

15. $1 - a^2 + 14ab - 49b^2$

$= 1 - (a^2 - 14ab + 49b^2)$

$= 1 - (a^2 - 2.7b.a + (7b)^2)$

$= 1^2 - (a - 7b)^2 = (1 - (a - 7b)).(1 + (a - 7b))$

$= (7b - a + 1)(a - 7b + 1)$ **Ans.**

16. $\frac{0.77 \times 0.77 \times 0.77 + 0.23 \times 0.23 \times 0.23}{0.77 \times 0.77 - 0.77 \times 0.23 + 0.23 \times 0.23}$

Let $x = 0.77$, $y = 0.23$

$\therefore \frac{x^3 + y^3}{x^2 + xy + y^2} = \frac{(x+y)(x^2 - xy + y^2)}{x^2 - xy + y^2}$

$= x + y = 0.77 + 0.23 = 1$ **Ans.**

17. P.T.

$(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a) = 2(a^3 + b^3 + c^3 - 3abc)$

Let $x = a + b$, $y = b + c$, $z = c + a$

L.H.S. $= x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z).\left(\frac{(x-y)^2 + (y-z)^2 + (z-x)^2}{2}\right)$

$= \frac{(a+b+b+c+c+a)}{2}.((a+b-b-c)^2 + (b+c-c-a)^2 + (c+a-a-b)^2)$

$= \frac{2(a+b+c)}{2}((a-c)^2 + (b-a)^2 + (c-b)^2)$

$= (a+b+c).((a-c)^2 + (a-b)^2 + (b-c)^2)$

$= 2.\frac{(a+b+c)}{2}.[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$= 2.(a^3 + b^3 + c^3 - 3abc)$

18. (i) $x^6 - y^6$

$= (x^3)^2 - (y^3)^2$

$= (x^3 - y^3).(x^3 + y^3)$

$= (x - y)(x^2 + xy + y^2).(x + y).(x^2 - xy + y^2)$

$= (x - y)(x + y)(x^2 + xy + y^2).(x^2 - xy + y^2)$

(ii) $x^{12} - y^{12}$

$= (x^6)^2 - (y^6)^2$

$= (x^6 - y^6).(x^6 + y^6)$

$= [(x^3)^2 - (y^3)^2] \cdot [(x^3)^2 + (y^3)^2]$

$= (x^3 - y^3)(x^3 + y^3)(x^2 + y^2).(x^4 - x^2y^2 - y^4)$

$= (x - y)(x + y).(x^2 + xy + y^2).(x^2 - xy + y^2).$

$(x^2 + y^2).(x^4 - x^2.y^2 + y^4)$ **Ans.**

19. $x^4 + \frac{1}{x^4} = 47$; $x^3 + \frac{1}{x^3} = ?$

$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 = 47 + 2$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 49$

$x^2 + \frac{1}{x^2} = \pm 7$

$\therefore x^2 + \frac{1}{x^2} = 7$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 9$

$\Rightarrow x + \frac{1}{x} = \pm 3$

$x + \frac{1}{x} = 3$ ...(1)

Cubic on both side

$\left(x + \frac{1}{x}\right)^3 = 3^3$

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$

From Eqⁿ (1)

$x^3 + \frac{1}{x^3} + 3(3) = 27$

$x^3 + \frac{1}{x^3} = 27 - 9 = 18$

$\boxed{x^3 + \frac{1}{x^3} = 18}$ **Ans.**

$x + \frac{1}{x} = -3$

Cubic on both sides

$\left(x + \frac{1}{x}\right)^3 = (-3)^3$

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = -27$

$\left(x^3 + \frac{1}{x^3}\right) + 3(-3) = -27$

$x^3 + \frac{1}{x^3} - 9 = -27$

$\boxed{x^3 + \frac{1}{x^3} = -18}$ **Ans.**

20. $a + b = 10$ & $a^2 + b^2 = 58$, $a^3 + b^3 = ?$

$a^3 + b^3 = (a + b).(a^2 - ab + b^2)$

$= (10)(58 - 21)$

$= 370$ **Ans.**

$(a + b)^2 = a + b^2 + 2ab$

$10^2 = a^2 + b^2 + 2(ab)$

$10^2 = 58 + 2(2b)$

$ab = \frac{100 - 58}{2}$

$ab = \frac{42}{2}$

$ab = 21$

21. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, P.T. $a = b = c$.

$\downarrow$

$2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

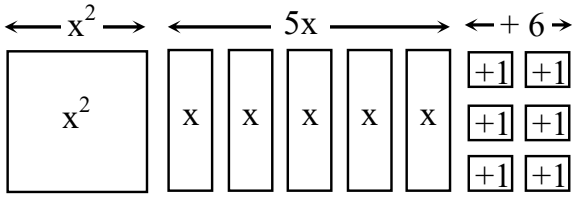
$\Rightarrow (a - b) = 0, b - c = 0$ & $c - a = 0$

$\Rightarrow \boxed{a = b = c}$ H.P.

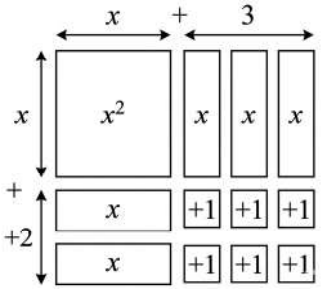
22. Given

$x^2 + 5x + 6$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$x^2 + 5x + 6$

$= (x + 3)(x + 2)$

➢ Long Answer Type Questions

23. Let $x = a^2 - b^2$, $y = b^2 - c^2$ and $z = c^2 - a^2$

Now, $x + y + z = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$x^3 + y^3 + z^3 = 3xyz = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$= 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)$

Also let p = a – b, q = b – c and r = c – a
Now, p + q + r = a – b + b – c + c – a = 0
$\therefore p^3 + q^3 + r^3 = 3pqr$
$\Rightarrow (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$
Hence,

$$\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$

$$= \frac{3(a+b)(a-b)(b+c)(b-c)(c+a)(c-a)}{3(a-b)(b-c)(c-a)}$$

$= (a + b)(b + c)(c + a)$

24.

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab} = \frac{a(b+c)^2 + b(c+a)^2 + c(a+b)^2}{abc}$$

$$= \frac{a(-a)^2 + b(-b)^2 + c(-c)^2}{abc} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc} = 3$$

[$\because$ If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$]

25. Let a = 2x – 5y, b = 2x + 5y
Use identity $a^3 - b^3$

26. $(-z + x - 2y).(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz)$
$= (x - 2y - z).((x)^2 + (2y)^2 + (-z)^2 + x(2y) - (x)(-z)) - (-2y)(-z)$
Let p = x, q = –2y, r = –z
$= (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$
$= p^3 + q^3 + r^3 - 3pqr$
$= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z)$
$= x^3 - 8y^3 - z^3 - 6xyz$ **Ans.**

27. Given that, a + b + c = 0 P.T.

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$

$$\text{L.H.S} = \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

$$= \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc}$$

$$= 3 = \text{RHS} \quad \left(\because a^3 + b^3 + c^3 = 3abc \text{ if } a + b + c = 0\right)$$

or

$$\text{L.H.S} = \frac{a^3 + b^3 + c^3}{abc} = \frac{(-b-c)^3 + b^3 + c^3}{abc}$$

$$= \frac{(b^3 + c^3 + 3bc(b+c)) + b^3 + c^3}{abc}$$

$$= \frac{-3bc(b+c)}{abc}$$

$$= \frac{-3bc(-a)}{abc}$$

$= 3$ **Ans.**

28. Given that,

$$\left.\begin{matrix} a + b + c = 5 \\ \& \\ ab + bc + ca = 10 \end{matrix}\right] \quad \ldots(1)$$

Now,
$a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
As we know that,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 5^2 = (a^2 + b^2 + c^2) + 2(10)$
$\Rightarrow a^2 + b^2 + c^2 = 5$ …(2)
From Eq (1) & (2),
$a^3 + b^3 + c^3 - 3ab$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= (5)(5 - (10))$
$= -25$
= R.H.S

29. P.T. $(a + b + c)^3 - a^3 - b^3 - c^3$
$= 3(a + b)(b + c)(c + a)$
L.H.S $= (a + b + c)^3 - a^3 - b^3 - c^3$
$q(a, b, c) = (a + b + c)^3 - a^3 - b^3 - c^3$
$= c^3 - c^3 = 0$
Put a = –b then
$q(-b, b, c) = (-b + b + c)^3 - (-b)^3 - b^3 - c^3$
$= c^3 - c^3 = 0$
$\therefore$ (a + b) is a factor of q(a, b, c)
Put b = –c, then
$q(a, -c, c) = (a - c + c)^3 - a^3 - (-c)^3 - c^3$
$= a^3 - a^3 = 0$
$\therefore$ (b + c) is a factor of q(a, b, c)
Put c = –a, then
$q(a, b, -a) = (a + b - a)^3 - a^3 - b^3 - (-a)^3$
$= b^3 - a^3 - b^3 + a^3 = 0$
$\therefore$ (c + a) is a factor of q(a, b, c)

Therefore,

$(a+b+c)^3 - (a^3+b^3+c^3)$

$= k(a+b)(b+c)(c+a)$

To find the value of k put a = 1, b = +1, c = 0

$(1+1+0)^3 - (1^3+1^3+0^3)$

$= k(1+1)(1+0)(0+1)$

$8 - 2 = k.(2).1.1$

$\Rightarrow 2k = 6 \qquad \Rightarrow \boxed{k = 3}$

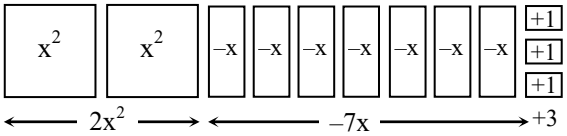
$\therefore (a+b+c)^3 - (a^3+b^3+c^3)$

$= 3(a+b)(b+c)(c+a)$ H.P.

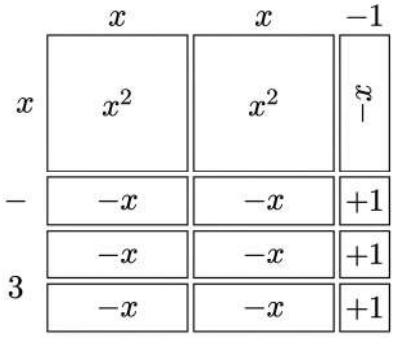
30. Given

$2x^2 - 7x + 3$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$2x^2 - 7x + 3$

$= (x-3)(2x-1)$

EXERCISE-2

1.[B] $11a^2 - 12\sqrt{2}a + 2$

$= 11a^2 - 11\sqrt{2}a - \sqrt{2}a + 2$

$= 11a(a - \sqrt{2}) - \sqrt{2}(a - \sqrt{2})$

$= (11a - \sqrt{2}).(a - \sqrt{2})$

2.[A] Let, $p(x) = x^5 - a^2x^3 - x^2y^3 + a^2y^3$

$= x^3(x^2 - a^2) - y^3(x^2 - a^2)$

$= (x^3 - y^3)(x^2 - a^2)$

$= (x-y).(x^2+y^2+xy).(x-a)(x+a)$

$= (x-a).(x+a).(x-y).(x^2+y^2+xy)$

3.[A] $a^3 + b^3 + 3ab - 1$

$= a^3 + b^3 + (-1)^3 - 3(a)(b)(-1)$

[Using the identity $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c).(a^2+b^2+c^2-ab-bc-ca)$]

$= (a+b-1)(a^2+b^2+1-ab-b(-1)-(-1)(a)$

$= (a+b-1).(a^2+b^2+1-ab+b+a)$

4.[D] $\sqrt{\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}}$

$= \sqrt{\left(\frac{x}{3}\right)^2 + \left(\frac{3}{2x}\right)^2 - \left(\frac{3}{2}\right)^2 + 2\left(\frac{x}{3}\right)\left(-\frac{3}{2x}\right) + 2\left(-\frac{3}{2x}\right)}$

$= \sqrt{\left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)^2 + \left(\frac{1}{2}\right)^2 + 2\left(\frac{x}{3}\right)\left(\frac{3}{2x}\right) + 2\left(\frac{3}{2x}\right)\left(-\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right)\left(\frac{x}{3}\right)}$

[Using the identity,

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$]

$= \sqrt{\left(\frac{x}{3} + \frac{3}{2x} - \frac{1}{2}\right)^2}$

$= \frac{x}{3} + \frac{3}{2x} - \frac{1}{2}$

5.[C]

$$\begin{array}{r|l} & 2x^2+3x+4 \\ \hline 2x^2+2x^2 & 4x^4+12x^3+25x^2+24x+16 \\ & \underline{-\,4x^4} \\ \hline 4x^2+3x & 12x^3+25x^2+24x+16 \\ +3x & \underline{-\,12x^3+\;\;9x^2} \\ \hline 4x^2+6x & 16x^2+24x+16 \\ +4 & \underline{-\;\;16x^2-\;\;24x-\;\;16} \\ & 0 \end{array}$$

$\therefore \sqrt{4x^4+12x^3+25x^2+24x+16} = 2x^2+3x+4$

$\therefore a = 2, b = 3, c = 4$

6.[B] If $a + b + c = 6$, then

$(2-a)^3 + (2-b)^3 + (2-c)^3 - 3(2-a)(2-b)(2-c) = ?$

Let $2 - a = x$

$2 - b = y$

$\Rightarrow x + y + z = (2-a)(2-b) + (2-c)$

$2 - c = z$

$\Rightarrow x + y + z = 6 - (a+b+c)$

$\Rightarrow x + y + z = 6 - 6 = 0 \quad \ldots (1)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz$

$= (x+y+z).(x^2+y^2+z^2-xy-yz-zx) = 0$

7.[A] Given that,

$$x + \frac{1}{x} = 2 \qquad \ldots (1)$$

then, $x^6 + \frac{1}{x^6} = ?$

First, do squaring on both sides,

$\left(x + \frac{1}{x}\right)^2 = 2^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 2 \quad \ldots (2)$

Now, Take cubic on both sides, we get,

$\left(x^2 + \frac{1}{x^2}\right)^3 = 2^3$

$\Rightarrow x^6 + \frac{1}{x^6} + 3(x)^2 + \left(\frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right) = 8$

$\Rightarrow x^6 + \frac{1}{x^6} + 3(2) = 8$

$\Rightarrow x^6 + \frac{1}{x^6} = 8 - 6$

$\Rightarrow \boxed{x^6 + \frac{1}{x^6} = 2}$ **Ans.**

Method 2 : Given that ; $x + \frac{1}{x} = 2$

$x + 1 = 2x$

$\Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$

Now, $x^6 + \frac{1}{x^6} = 1^6 + \frac{1}{1^6}$

$\Rightarrow x^6 + \frac{1}{x^6} = 2$ **Ans.**

8.[C] Given that,

$x + y + z = 6, \quad \ldots (1)$

$xy + yz + zx = 11, \quad \ldots (2)$

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z).(x^2 + y^2 + z^2 - xy - yz - zx)$

From Eq^n (1),

$(x + y + z)^2 = 6^2$

$(x^2 + y^2 + z^2) + 2(xy + yz + zx) = 36$

From Eq^n (1) & (2)

$(x^2 + y^2 + z^2) + 2(11) = 36$

$x^2 + y^2 + z^2 = 36 - 22$

$\boxed{x^2 + y^2 + z^2 = 14} \quad \ldots (3)$

$\therefore x^3 + y^3 + z^3 - 3xyz = (6).(14 - (11))$

$= 6 \times (3) = 18$

9.[A] $\frac{a^3 + b^3 + c^3 - 3abc}{-(a^2 + b^2 + c^2 - ab - bc - ca)}$

$= \frac{(a + b + c).(a^2 + b^2 + c^2 - ab - bc - ca)}{-(a^2 + b^2 + c^2 - ab - bc - ca)}$

Given that, $\begin{cases} a = -5 \\ b = -6 \\ c = 10 \end{cases}$

$= -(a + b + c) = -(-5 - 6 + 10)$

$= -(-11 + 10) = 1$

10.[D] Given that,

$a^2 - b^2 = 21$

$a^2 + b^2 = 29$

$2a^2 = 50$

$a^2 = 25$

$\boxed{a = \pm 5}$

$a^2 + b^2 = 21$

$25 - b^2 = 21$

$b^4 = 4$

$\boxed{b = \pm 2}$

$\therefore ab = \pm 10$

11.[D] $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$

$= (a + b)^3 + (a - b)^3 + 3((a + b)) + (a - b)(a + b)(a - b)$

$= [(a + b) + (a - b)]^3 = (2a)^3 = 8a^3$

12.[B] Given that,

$\left.\begin{array}{c} x + y + z = 1 \\ xy + yz + zx = -1 \\ xyz = -1 \end{array}\right\} \quad \ldots (1)$

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

From (1),

$(x + y + z)^2 = 1^2$

$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 1$

$(x^2 + y^2 + z^2) - 2(-1) = 1$

$x^2 + y^2 + z^2 = 3$

$\therefore x^3 + y^3 + z^3 - 3(-1) = (1).(3 - (-1))$

$\Rightarrow x^3 + y^3 + z^3 + 3 = 4$

$x^3 + y^3 + z^3 = 1$

13.[D] $\frac{(a - b)^2}{(b - c)(c - a)} + \frac{(b - c)^2}{(a - b)(c - a)} + \frac{(c - a)^2}{(a - b)(b - c)}$ L

$\left.\begin{array}{ll} & a - b = x \\ \text{et} & b - c = y \\ & c - a = z \end{array}\right]$

$\therefore \quad x + y + z = 0$

$\therefore \quad x^3 + y^3 + z^3 - 3xyz = 0$

$x^3 + y^3 + z^3 = 3xyz$

$= \dfrac{(a-b)^3 + (b-c)^3 + (c-a)^2}{(a-b)(b-c)(c-a)}$

$= \dfrac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

14.[B] $\dfrac{0.76\times0.76\times0.76 + 0.24\times0.24\times0.24}{0.76\times0.76 - 0.76\times0.24 + 0.24\times0.24}$

Let x = 0.76, y = 0.24

$= \dfrac{x^3 + y^3}{x^2 - xy + y^2}$

$= \dfrac{(x+y).(x^2 - xy + y^2)}{x^2 - xy + y^2} = x + y = 0.76 + 0.24 = 1$

15. [D] 3xy

$(x + y)^3 - (x^3 + y^3)$

$\therefore (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= [x^3 + 3x^2y + 3xy^2 + y^3]$
$- [(x + y)(x^2 - xy + y^2)]$

$= [x^3 + 3x^2y + 3xy^2 + y^3]$
$- [x(x^2 - xy + y^2) + y(x^2 - xy + y^2)]$

$= [x^3 + 3x^2y + 3xy^2 + y^3]$
$- [x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3]$

$= [x^3 + 3x^2y + 3xy^2 + y^3] - [x^3 + y^3]$

$= x^3 + 3x^2y + 3xy^2 + y^3 - x^3 - y^3$

$= 3x^2y + 3xy^2$

$= 3xy(x + y)$

16. [C] 0

$\dfrac{x}{y} + \dfrac{y}{x} = -1$

$\Rightarrow \dfrac{x^2 + y^2}{xy} = -1$

$\Rightarrow x^2 + y^2 = -xy$

$\Rightarrow x^2 + y^2 + xy = 0 \quad \ldots(i)$

$x^3 - y^3$

$\therefore a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (x - y)(x^2 + xy + y^2)$

from eq. (i)

$= (x - y)(0)$

$= 0$

17.[C] $\dfrac{1}{4}$

$49x^2 - b = \left(7x + \dfrac{1}{2}\right)\left(7x - \dfrac{1}{2}\right)$

$\therefore (a + b)(a - b) = a^2 - b^2$

$\Rightarrow 49x^2 - b = (7x)^2 - \left(\dfrac{1}{2}\right)^2$

$\Rightarrow 49x^2 - b = 49x^2 - \dfrac{1}{4}$

Compare both sides

$b = \dfrac{1}{4}$

18.[C] 5

$x^2 + kx + 6 = (x + 2)(x + 3)$

$\Rightarrow x^2 + kx + 6 = x(x + 3) + 2(x + 3)$

$\Rightarrow x^2 + kx + 6 = x^2 + 3x + 2x + 6$

$\Rightarrow x^2 + kx + 6 = x^2 + 5x + 6$

Compare both sides

k = 5

19.[C] $\sqrt{2} - 1$

$p(x) = kx^2 - \sqrt{2}\,x + 1$

g(x) = (x – 1)

Put g(x) = 0

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Put x = 1 in p(x) and p(x) = 0

$\Rightarrow p(1) = 0$

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

20.[A] $-\dfrac{1}{2}$

$p(x) = 10x^2 + px + q$

g(x) = (x + 2)

Put g (x) = 0

x + 2 = 0

$\Rightarrow x = -2$

Put x = – 2 in p(x) and p(x) = 0

$\Rightarrow p(-2) = 0$

$\Rightarrow 10(-2)^2 + p(-2) + q = 0$

$\Rightarrow 40 - 2p + q = 0$

$\Rightarrow 2p - q = 40 \quad \ldots(i)$

Now, g(x) = (x – 1)

Put g(x) = 0

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Put x = – 1 in p(x) and p(x) = 0

$\Rightarrow p(1) = 0$

$\Rightarrow 10(1)^2 + p(1) + q = 0$

$\Rightarrow 10 + p + q = 0$

$\Rightarrow p + q = -10$ …(ii)

eqn. (i) + eqn. (ii)

$\Rightarrow 2p - q + p + q = 40 + (-10)$

$\Rightarrow 3p = 40 - 10$

$\Rightarrow 3p = 30$

$\Rightarrow p = \frac{30}{3} = 10$

Put value of p in eqn. (ii)

$\Rightarrow p + q = -10$

$\Rightarrow 10 + q = -10$

$\Rightarrow q = -10 - 10$

$\Rightarrow q = -20$

$\frac{p}{q}$

$= \frac{10}{-20}$

$= -\frac{1}{2}$

21.[C] $\frac{x-5}{x^{1/2}}$

EXERCISE-3

1.[A] Given that,

$x + \frac{1}{x} = a + b$ … (1)

Adding (+) $x - \frac{1}{x} = a - b$ … (2)

$2x = 2a$

$\Rightarrow \boxed{x = a}$

From Eq. (1)

$x + \frac{1}{x} = a + b$

$a + \frac{1}{x} = a + b$

$\boxed{x = \frac{1}{b}}$

$\therefore x = a = \frac{1}{b}$

$\Rightarrow \boxed{ab = 1}$

2.[B] H.C.F. × L.C.M. = Product of two polynomials

$A.(x^3 - 1) = (x - 1).(x^6 - 1)$

$A = \frac{(x-1).(x^3-1)(x^3+1)}{(x^3-1)}$

$A = x^4 + x - x^3 - 1$

$A = x^4 - x^3 + x - 1$

3.[D] Let, $p(x) = x^2 + 5x + 6 = (x + 3).(x + 2)$

$q(x) = x^3 + 27 = (x + 3).(x^2 + 3x + 9)$

$\therefore$ H.C.F. of p(x) & q(x) = (x + 3)

4.[A] $(2x - 3y)^2 - 7(2x - 3y) - 30$

Let $2x - 3y = m$

$m^2 - 7m - 30 = m^2 - 10m + 3m - 30$

$= m(m - 10) + 3(m - 10)$

$= (m + 3)(m - 10)$

$= (2x - 3y + 3).(2x - 3y - 10)$

5.[B] Let

$p(x) = x^3 + x^2 + x + 1$

$= x^2(x + 1) + 1(x + 1)$

$= (x + 1).(x^2 + 1)$

$q(x) = x^3 - x^2 + x - 1$

$= x^2(x - 1) + 1(x - 1) = (x^2 + 1).(x - 1)$

$\therefore$ L.C.M. $(p(x), q(x)) = (x + 1)(x - 1)(x^2 + 1)$

$= (x^2 - 1).(x^2 + 1) = x^4 - 1$

6.[D] $x + \frac{1}{x} = 3$

Squaring on both sides, $\left(x + \frac{1}{x}\right)^2 = 9$

$x^2 + \frac{1}{x^2} + 2 = 9$

$x^2 + \frac{1}{x^2} = 7$

Taking cube on both sides, we get,

$\left(x^2 + \frac{1}{x^2}\right)^3 = 7^3$

$\Rightarrow x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 343$

$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 3.7$

$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 21 = 322$

7.[D] Given that, $a^5 = b^6 \Rightarrow a = b^{6/5}$ … (1)

$c^3 = d^4 \Rightarrow d = c^{3/4}$ … (2)

& $d - a = 61$ … (3)

From (1) & (2),

$c^{3/4} - b^{6/5} = 61$

($\because$ a, b, c & d are the natural numbers)

$\therefore$ c = 625 & b = 32

$\therefore$ c – b = 625 – 32 = 593

8.[C] Given that,

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$$

$$=\frac{1}{1+\left(\frac{x^b}{x^a}\right)+\left(\frac{x^c}{x^a}\right)}+\frac{1}{1+\left(\frac{x^a}{x^b}\right)+\left(\frac{x^c}{x^b}\right)}+\frac{1}{1+\left(\frac{x^b}{x^c}\right)+\left(\frac{x^a}{x^b}\right)}$$

$$=\left(\frac{x^a}{x^a+x^b+x^c}\right)+\left(\frac{x^b}{x^a+x^b+x^c}\right)+\left(\frac{x^c}{x^a+x^b+x^c}\right)$$

$$=\frac{x^a+x^b+x^c}{x^a+x^b+x^c}=1$$

9.[B] $p(x) = ax^3 + x^2 - 2x + 4a - 9$

$g(x) = (x + 1)$

Put $g(x) = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

Put $x = -1$ in $p(x)$ and $p(x) = 0$

$\Rightarrow p(-1) = 0$

$\Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$

$\Rightarrow -a + 1 + 2 + 4a - 9 = 0$

$\Rightarrow 3a - 6 = 0$

$\Rightarrow 3a = 6$

$\Rightarrow a = \frac{6}{3} = 2$

10.[B] ± 9

$x - y = 1, \quad x^2 + y^2 = 41$

$x - y = 1$

Square both side

$\Rightarrow (x - y)^2 = (1)^2$

$\Rightarrow x^2 + y^2 - 2xy = 1$

$\Rightarrow 41 - 2xy = 1$

$\Rightarrow 2xy = 41 - 1$

$\Rightarrow 2xy = 40$

$\Rightarrow xy = \frac{40}{2} = 20$

$(x + y)^2 = x^2 + 2xy + y^2$

$\Rightarrow (x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow (x + y)^2 = 41 + 2(20)$

$\Rightarrow (x + y)^2 = 41 + 40$

$\Rightarrow (x + y)^2 = 81$

Square root both sides

$\Rightarrow \sqrt{(x+y)^2} = \sqrt{81}$

$\Rightarrow x + y = \pm 9$

11.[A] $x^4 + \frac{1}{x^4} = 47$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x)^2\left(\frac{1}{x^2}\right) - 2(x)^2\left(\frac{1}{x^2}\right) = 47$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 - 2 = 47$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 47 + 2$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 49$$

Square root both sides

$$\Rightarrow \sqrt{\left(x^2+\frac{1}{x^2}\right)^2} = \sqrt{49}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 7$$

$$\Rightarrow (x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) - (2x)\left(\frac{1}{x}\right) = 7$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 - 2 = 7$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 7 + 2$$

Square root both sides

$$\Rightarrow \sqrt{\left(x+\frac{1}{x}\right)^2} = \sqrt{9}$$

$$\Rightarrow x + \frac{1}{x} = 3$$

12.[C] –8

$(a + b + c) = 4$

Square both sides

$\Rightarrow (a + b + c)^2 = (4)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + ca = 16$

$\Rightarrow a^2 + b^2 + c^2 + 2 (ab + bc + ca) = 16$

$\Rightarrow a^2 + b^2 + c^2 + 2 (6) = 16$

$\Rightarrow a^2 + b^2 + c^2 + 12 = 16$

$\Rightarrow a^2 + b^2 + c^2 = 16 - 12$

$\Rightarrow a^2 + b^2 + c^2 = 4$

$a^3 + b^3 + c^3 - 3abc$

$\therefore x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= (4) [4 - (ab + bc + ca)]$

$= 4 [4 - 6]$

$= 4 (-2)$

$= -8$

13. [C] $\ell + m + n = 0$

$$\frac{\ell^2}{mn} + \frac{m^2}{n\ell} + \frac{n^2}{\ell m}$$

$$= \frac{\ell^3 + m^3 + n^3}{\ell mn}$$

If $\ell + m + n = 0$, then $\ell^3 + m^3 + n^3 = 3\ell mn$

$$= \frac{3\ell mn}{\ell mn}$$

$= 3$

14.[C] $a + b + c = 5$, and $ab + bc + ca = 10$

$a + b + c = 5$

Square both sides

$\Rightarrow (a + b + c)^2 = (5)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 25$

$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$

$\Rightarrow a^2 + b^2 + c^2 + 2(10) = 25$

$\Rightarrow a^2 + b^2 + c^2 + 20 = 25$

$\Rightarrow a^2 + b^2 + c^2 = 25 - 20$

$\Rightarrow a^2 + b^2 + c^2 = 5$

$a^3 + b^3 + c^3 - 3abc$

$\therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)$

$(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= (5) [a^2 + b^2 + c^2 - (ab + bc + ca)]$

$= (5) [5 - (10)]$

$= 5 [-5]$

$= -25$

Sol.15 [A]

$a^2 + b^2 + c^2 = 90$, $\quad a + b + c = 20$

$a + b + c = 20$

square both sides

$\Rightarrow (a + b + c)^2 = (20)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 400$

$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx)$

$\Rightarrow 90 + 2(ab + bc + ca) = 400$

$\Rightarrow 2(ab + bc + ca) = 400 - 90$

$$\Rightarrow ab + bc + ca = \frac{310}{2}$$

$\Rightarrow ab + bc + ca = 155$

16.[D] $\dfrac{(x+5)}{(x+1)}$

$$\frac{x^2 + 4x - 5}{x^2 - 1}$$

$$= \frac{x^2 + 4x - 5}{(x)^2 - (1)} \qquad \therefore a^2 - b^2 = (a + b)(a - b)$$

$$= \frac{x^2 + 5x - x - 5}{(x+1)(x-1)}$$

$$= \frac{x(x+5) - 1(x+5)}{(x+1)(x-1)}$$

$$= \frac{(x+5)(x-1)}{(x+1)(x-1)}$$

$$= \frac{(x+5)}{(x+1)}$$

MENTAL ABILITY

Chapter

1

NUMBER SERIES

Mental ability is the ability of mind to observe and understand things or patterns in a logical way and reach to a conclusion or judgment based on that logic.

It is the ability to distinguish between important, less important and more important.

Number Series problems deal with numbers. While attempting to solve the question, you have to check the pattern of the series. Series moves with certain mathematical operations like :

- Consecutive odd/even numbers.
- Consecutive prime / composite numbers.
- Squares/cubes of some numbers with/without variation of addition or subtraction of some number.
- Sum/product/difference of preceding number(s) .
- Addition/subtraction/multiplication/division by some number.
- Many more combinations of the relationship given above.

Type of questions asked in the examination:

(i) Find the missing term(s).

(ii) Find the wrong term(s).

◈ FIND THE MISSING TERM

Ex.1 7, 12, 19, ?, 39

(A) 29 (B) 28 (C) 26 (D) 24

Sol.(B) The given sequence follows the pattern:

+5, +7, +9 ... i.e., 7 + 5 = 12, 12 + 7 = 19, ...

∴ Missing number = 19 + 9 = 28

Ex.2 0, 6, 24, 60, 120, 210, ?

(A) 240 (B) 290 (C) 336 (D) 504

Sol.(C) The given series is

$1^3 - 1, 2^3 - 2, 3^3 - 3, 4^3 - 4, 5^3 - 5, 6^3 - 6.$

∴ Next number = $7^3 - 7 = 343 - 7 = 336$

Ex.3 4, 6, 12, 14, 28, 30, ?

(A) 32 (B) 60 (C) 62 (D) 64

Sol.(B) The given sequence is a combination of two series :

I. 4, 12, 28, ? and II. 6, 14, 30.

Now, the pattern followed in each of the above two series is : +8, +16, +32 ....

So, missing number = (28 + 32) = 60

Ex.4 1, 3, 3, 6, 7, 9, ?, 12, 21.

(A) 10 (B) 11 (C) 12 (D) 13

Sol.(D) The given sequence is a combination of two series :

I. 1, 3, 7, ?, 21 and II. 3, 6, 9, 12

The pattern followed in I is + 2, + 4, +6, +8 ... and the pattern followed in II is +3. Thus, missing number = 7 + 6 = 13.

Ex.5 $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ?$

(A) $\frac{9}{32}$ (B) $\frac{10}{17}$ (C) $\frac{11}{34}$ (D) $\frac{12}{35}$

Sol.(A) Clearly, the numerators of the fractions in the given sequence form the series 1, 3, 5, 7, in which each term is obtained by adding 2 to the previous term. The denominators of the fractions form the series 2, 4, 8, 16

i.e. $2^1, 2^2, 2^3, 2^4$.

So, the numerator of the next fraction will be (7 + 2) i.e., 9 and the denominator will be 2^5 i.e. 32.

∴ The next term is $\frac{9}{32}$

Ex.6 94, 166, 258, ?, 4912

(A) 3610 (B) 2490

(C) 789 (D) 810

Sol.(A) Each number is in two parts. The first part is square of consecutive numbers 3, 4, 5, ……..

$(3)^2$ $(4)^2$ $(5)^2$ $(6)^2$ $(7)^2$

9 4 16 6 25 8 36 10 49 12

4 6 8 10 12

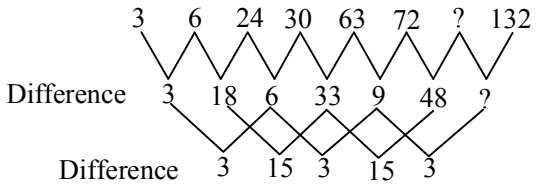
The second part is the sequence of number with difference +2, like 4, 6, 8, …….

Hence, the required number is 3610.

Ex.7 3, 6, 24, 30, 63, 72, ?, 132

(A) 128 (B) 122 (C) 120 (D) 124

Sol.(C) The difference between the terms is given below as:



Therefore, alternate difference between the difference is 3 and 15 respectively.

Hence, the next term would be 72 + 48 = 120.

Ex.8 6, 11, 21, 36, 56, ?

(A) 42 (B) 51 (C) 81 (D) 91

Sol.(C) The pattern is +5, +10, +15, + 20, ...

∴ Missing number = 56 + 25 = 81

Ex.9 1, 6, 13, 22, 33, ?

(A) 44 (B) 45 (C) 46 (D) 47

Sol.(C) The pattern is + 5, + 7, + 9, + 11, ...

∴ Missing number = 33 + 13 = 46

Ex.10 1, 9, 17, 33, 49, 73, ?

(A) 97 (B) 98 (C) 99 (D) 100

Sol.(A) The pattern is +8, +8, +16, +16, +24, ...

∴ Missing number = 73 + 24 = 97

Ex.11 3, 7, 15, 31, 63, ?

(A) 92 (B) 115 (C) 127 (D) 131

Sol.(C) Each number in the series is the preceding number multiplied by 2 and then increased by 1.

Thus, (3 × 2) + 1 = 7, (7 × 2) + 1 = 15,

(15 × 2) + 1 = 31 and so on.

∴ Missing number = (63 × 2) + 1 = 127

Ex.12 1, 6, 15, ?, 45, 66, 91

(A) 25 (B) 26 (C) 27 (D) 28

Sol.(D) The pattern is +5, +9, ..., +21, +25

∴ Missing number = 15 + 13 = 28

Ex.13 1, 2, 3, 5, 8, ?

(A) 9 (B) 11 (C) 13 (D) 15

Sol.(C) Each term in the series is the sum of the preceding two terms.

Thus, 1 + 2 = 3; 2 + 3 = 5; 3 + 5 = 8 and so on.

∴ Missing number = 5 + 8 = 13

Ex.14 77, 49, 36, 18, ?

(A) 8 (B) 9 (C) 4 (D) 12

Sol.(A) Next term in the series is the product of the digits of the previous term

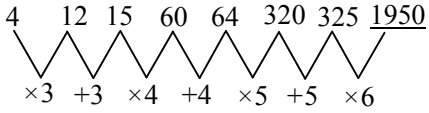
7 × 7 = 49, 4 × 9 = 36, 3 × 6 = 18,

Thus, 1 × 8 = 8

Ex.15 4, 12, 15, 60, 64, 320, 325, ?

(A) 1950 (B) 1850

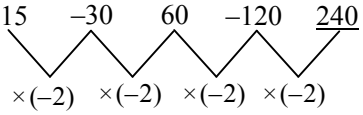
(C) 1935 (D) 1955

Sol.(A) 

Ex.16 15, –30, 60, –120, ?

(A) 150 (B) 240 (C) 270 (D) 180

Sol.(B)



◈ FIND THE WRONG TERM

Ex.17 2, 5, 9, 11, 14

(A) 2 (B) 5 (C) 9 (D) 11

Sol. (C) Series : + 3, + 3, + 3, ...

The next term is got by adding 3 in the preceding term.

$\therefore$ 2 + 3 = 5, 5 + 3 = 8

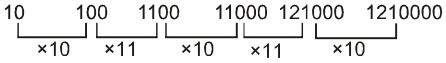
Hence, 9 is the wrong term.

Ex.18 10, 100, 1100, 11000, 111000, 1210000.

(A) 1210000 (B) 11000

(C) 100 (D) 111000

Sol. (D) Given series is :

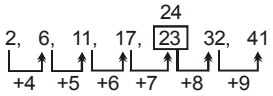


Hence, the wrong term is 111000.

Ex.19 2, 6, 11, 17, 23, 32, 41

(A) 6 (B) 17 (C) 23 (D) 32

Sol. (C) Given series is :



Hence, the wrong term is 23.

Ex.20 61, 52, 63, 95, 46, 18

(A) 95 (B) 63 (C) 46 (D) 52

Sol. (A) On interchanging the digits of each term, we get a number which is a perfect square of a natural number.

$4^2 = 16 \Rightarrow 61$, $5^2 = 25 \Rightarrow 52$, $6^2 = 36 \Rightarrow 63$,

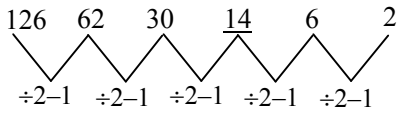
$7^2 = 49 \Rightarrow 94$,

Hence, the wrong term is 95.

Ex.21 126, 62, 30, 15, 6, 2

(A) 15 (B) 30 (C) 6 (D) 62

Sol. (A)



Hence, the wrong term is 15.

EXERCISE-1

Directions: (Q.1 to Q.15) Find the missing term.

Q.1 9, 7, 14, 11, 33, 28, ?, 133, 931
(A) 64 (B) 46 (C) 140 (D) 123

Q.2 6, 12, 23, 44, 85, ?
(A) 166 (B) 174 (C) 165 (D) 168

Q.3 92, 23, 27, 9, 12, 6, ?
(A) 20 (B) 16 (C) 8 (D) 10

Q.4 0, 5, 8, 17, 24, 37, 48, ?
(A) 65 (B) 67 (C) 56 (D) 71

Q.5 999, 730, 510, 345, 213, ?
(A) 122 (B) 126 (C) 68 (D) 128

Q.6 2, 5, 14, 7, 3, 20, 10, 4, ?
(A) 26 (B) 30 (C) 22 (D) 28

Q.7 3, 7, 35, 47, ? , 119
(A) 99 (B) 98 (C) 64 (D) 89

Q.8 23, 27, 43, 79, 143, ?
(A) 242 (B) 241 (C) 190 (D) 243

Q.9 870, 342, 132, ? , 12, 2
(A) 90 (B) 42 (C) 64 (D) 66

Q.10 9, 19, 35, 75, 143, ?
(A) 287 (B) 285 (C) 196 (D) 295

Q.11 2, 20, 110, 380, ?
(A) 424 (B) 650 (C) 982 (D) 992

Q.12 256, 125, 60, ? 12.5
(A) 26 (B) 40 (C) 28 (D) 25

Q.13 6, 15, 35, 77, 143, 221, 323, ?
(A) 437 (B) 427 (C) 384 (D) 365

Q.14 9, 9, 14, 34, 124, ?
(A) 576 (B) 532 (C) 448 (D) 604

Q.15 125, 215, 343, 511, 729, ?
(A) 754 (B) 683 (C) 845 (D) 999

Directions: (Q.16 to Q.25) Find the wrong term.

Q.16 10, 26, 74, 218, 654, 1946, 5834
(A) 26 (B) 74 (C) 218 (D) 654

Q.17 3, 7, 15, 39, 63, 127, 255, 511
(A) 15 (B) 39 (C) 63 (D) 127

Q.18 1236, 2346, 3456, 4566, 5686
(A) 1236 (B) 3456 (C) 4566 (D) 5686

Q.19 10, 15, 26, 35, 48, 63, 82
(A) 48 (B) 26 (C) 63 (D) 82

Q.20 445, 221, 109, 46, 25, 11, 4
(A) 25 (B) 46 (C) 109 (D) 221

Q.21 2, 6, 10, 20, 30, 42, 56
(A) 6 (B) 10 (C) 20 (D) 30

Q.22 3, 9, 27, 82, 243
(A) 27 (B) 54 (C) 82 (D) 162

Q.23 320, 254, 200, 155, 122, 100, 89
(A) 155 (B) 320 (C) 254 (D) 200

Q.24 15, 34, 71, 134, 223, 350
(A) 71 (B) 134 (C) 223 (D) 350

Q.25 5, 10, 40, 80, 320, 550, 2560
(A) 80 (B) 320 (C) 550 (D) 2560

EXERCISE-2

(Previous Year Questions - NTSE & NSO)

Directions: (Q.1 to Q.52) Find the missing term

Q.1 2, 3, 10, 15, 26, ?
(A) 34 (B) 35 (C) 36 (D) 37

Q.2 1, 4, 27, 16, 125, 36, ?
(A) 216 (B) 343 (C) 64 (D) 49

Q.3 336, 210, 120, ?, 24, 6, 0
(A) 40 (B) 50 (C) 60 (D) 70

Q.4 3, 4, 8, 17, 33, ?
(A) 58 (B) 69 (C) 49 (D) 98

Q.5 8, 13, 21, 34, 55, ?
(A) 60 (B) 68 (C) 89 (D) 76

Q.6 480, 480, 240, 80, 20, ?
(A) 4 (B) 1 (C) 5 (D) 10

Q.7 1, 1, 2, 2, 3, 4, 4, 8, 5, 16, ?
(A) 6 (B) 32 (C) 8 (D) 7

Q.8 2, 5, 11, 23, 47, ?
(A) 92 (B) 90 (C) 95 (D) 91

Q.9 12, 21, 23, 32, 34, 43, 45, ?
(A) 54 (B) 48 (C) 77 (D) 9

Q.10 14, 1, 21, 4, 28, 9, ?, ?
(A) 9, 42 (B) 16, 35
(C) 35, 16 (D) 16, 36

Q.11 5, 6, 13, 26, 45, ?
(A) 68 (B) 74 (C) 70 (D) 82

Q.12 190, 94, 46, 22, 10, 4, ?
(A) 3 (B) 2 (C) 1 (D) 0

Q.13 128, 110, 90, 68, ?
(A) 236 (B) 42 (C) 44 (D) 48

Q.14 1, 2, 4, 7, ?, 16
(A) 9 (B) 11 (C) 12 (D) 13

Q.15 6, 8, 9, 12, 14, 18, ?
(A) 21 (B) 19 (C) 23 (D) 20

Q.16 4, 9, 19, 34, 54, ?
(A) 66 (B) 75 (C) 79 (D) 84

Q.17 31, 29, 24, 22, 17, ?, ?
(A) 15, 13 (B) 10, 8 (C) 14, 12 (D) 15, 10

Q.18 3, 6, 11, 18, ?
(A) 19 (B) 27 (C) 30 (D) 37

Q.19 3, 8, 15, 24, ?
(A) 30 (B) 35 (C) 36 (D) 49

Q.20 4, 10, 23, 50, 105, ?
(A) 215 (B) 210 (C) 216 (D) 439

Q.21 912, 303, 102, 33, ?, 3, 2
(A) 12 (B) 10 (C) 8 (D) 6

Q.22 1, 4, 9, ?, 25, 36
(A) 11 (B) 19 (C) 21 (D) 16

Q.23 7, 12, 22, 37, ?, 82, 112
(A) 62 (B) 57 (C) 52 (D) 42

Q.24 11, 13, 17, 19, ?, 25
(A) 20 (B) 21 (C) 23 (D) 22

Q.25 5, 9, 17, 33, ?, 129
(A) 72 (B) 67 (C) 65 (D) 58

Q.26 2, 5, 4, 10, 7, 15, 11, 20, ?, ?
(A) 12, 21 (B) 16, 25
(C) 13, 25 (D) 17, 30

Q.27 0, 6, 24, 60, 120,?
(A) 180 (B) 224 (C) 196 (D) 210

Q.28 57, 54, 58, 55, 59, 56, 60, ?
(A) 64 (B) 63 (C) 58 (D) 57

Q.29 27, 31, 40, 56, 81, 117, ?
(A) 156 (B) 165 (C) 166 (D) 169

Q.30 55, 168, 57, 120, 60, 80, 62, 48, 65, 24, ?, ?
(A) 69, 11 (B) 67, 8 (C) 8, 71 (D) 6, 72

Q.31 8, 7, 16, 5, 32, 3, 64, 1, 128, ?
(A) 18 (B) 13 (C) –1 (D) 3

Q.32 16, 33, 65, 131, ?, 523
(A) 261 (B) 521 (C) 613 (D) 721

Q.33 5, 2, 17, 4, ?, 6, 47, 8, 65
(A) 29 (B) 30 (C) 31 (D) 32

Q.34 1, 2, 4, 8, ?, 32
(A) 10 (B) 12 (C) 14 (D) 16

Q.35 2, 3, 10, 15, 26, ?
(A) 36 (B) 35 (C) 39 (D) 48

Q.36 2, 30, 6, 20, 12, 12, ?
(A) 26 (B) 22 (C) 20 (D) 24

Q.37 6, 20, 36, 48, 50, ?, 0
(A) 36 (B) 40 (C) 46 (D) 56

Q.38 7, 15,28, 59, 114, ?
(A) 243 (B) 233 (C) 213 (D) 223

Q.39 25, 49, 89, 145, 217, ?
(A) 305 (B) 327 (C) 309 (D) 303

Q.40 0, 2, 2, 3, 3, 5, 8, 4, 10, ?, 5, 17
(A) 6 (B) 7 (C) 9 (D) 15

Q.41 0, 2, 24, 252, ?
(A) 620 (B) 1040 (C) 3120 (D) 5430

Q.42 6, 24, 60, 120, ?
(A) 180 (B) 210
(C) 240 (D) 360

Q.43 2, 10, 26, ?, 242
(A) 80 (B) 81
(C) 82 (D) 84

Q.44 5, 10, 17, 26, 37, 50, ?
(A) 70 (B) 66 (C) 65 (D) 64

Q.45 6, 25, 62, 123, ?, 341
(A) 216 (B) 214 (C) 215 (D) 217

Q.46 5, 3, 10, 8, 17, 15, ?, 24
(A) 26 (B) 27 (C) 29 (D) 36

Q.47 2, 6, 12, 20, 30 ?
(A) 40 (B) 42 (C) 44 (D) 46

Q.48 12, 22, 69, 272, 1365, ?
(A) 8196 (B) 8184 (C) 8195 (D) 6830

Q.49 65, 48, 64, 49, 63, ? .
(A) 53 (B) 52 (C) 51 (D) 50

Q.50 7, 23, ? , 79, 119.
(A) 47 (B) 49 (C) 44 (D) 46

Q.51 16, 8, 12, ? , 105.
(A) 6 (B) 30 (C) 24 (D) 35

Q.52 748, 737, 716, 685, 644, ? .
(A) 634 (B) 643 (C) 503 (D) 593

Directions: (Q.53 to Q.56)
In each of the question **53** to **56** some of the numbers are missing in the given series with one term missing shown by question mark (?). This terms is one of the alternatives among the four numbers given under it. Find the right alternative.

Q.53 8, 27, 64, ?, 216, 343
(A) 125 (B) 81 (C) 100 (D) 196

Q.54 5, 11, 19, ? , 41
(A) 28 (B) 29 (C) 30 (D) 35

Q.55 120, ?, 24, 6, 0
(A) 100 (B) 70 (C) 60 (D) 20

Q.56 729, 81, 9, 1, $\frac{1}{9}$, ? , $\frac{1}{729}$
(A) $\frac{1}{27}$ (B) $\frac{1}{81}$ (C) $\frac{1}{243}$ (D) $\frac{1}{486}$

Directions: (Q.57 to Q.79) Find the wrong term

Q.57 3, 7, 9, 21, 27, 66, 81, 189, 243
(A) 27 (B) 66 (C) 243 (D) 21

Q.58 27, 34, 40, 45, 49, 53, 54, 55
(A) 53 (B) 45 (C) 56 (D) 34

Q.59 0, 2, 3,6, 6, 20, 9, 54, 12
(A) 3 (B) 6 (C) 20 (D) 54

Q.60 0, 2, 10, 36, 68, 130
(A) 10 (B) 36 (C) 68 (D) 130

Q.61 9, 54, 44, 264, 254, 1520, 1514
(A) 1514 (B) 1520 (C) 264 (D) 44

Q.62 10, 15, 26, 35, 48, 63, 82
(A) 48 (B) 26 (C) 63 (D) 82

Q.63 3, 10, 30, 66, 127, 218
(A) 3 (B) 66 (C) 30 (D) 218

Q.64 7, 9, 17, 42, 91, 172, 293
(A) 91 (B) 42 (C) 17 (D) 9

Q.65 2, 12, 24, 34, 68, 78, 158, 166
(A) 68 (B) 78 (C) 158 (D) 166

Q.66 2, 6, 10, 20, 30, 42, 56
(A) 6 (B) 10 (C) 20 (D) 30

Q.67 7, 9, 16, 25, 41, 68, 107, 173
(A) 16 (B) 41 (C) 68 (D) 107

Q.68 3, 9, 27, 82, 243
(A) 27 (B) 54 (C) 82 (D) 162

Q.69 5, 9, 17, 35, 65, 129
(A) 65 (B) 35 (C) 17 (D) 9

Q.70 1, 5, 6, 11, 17, 27, 45, 73
(A) 27 (B) 45 (C) 17 (D) 11

Q.71 3, 6, 11, 18, 28, 38, 51, 66
(A) 18 (B) 28 (C) 38 (D) 51

Q.72 320, 254, 200, 155, 122, 100, 89
(A) 155 (B) 320 (C) 254 (D) 200

Q.73 6, 8, 9, 12, 14, 18, 22, 26, 30
(A) 12 (B) 22 (C) 26 (D) 30

Q.74 3, 7, 9, 28, 27, 84, 81, 448, 243
(A) 84 (B) 81 (C) 28 (D) 7

Q.75 190, 94, 46, 22, 10, 3
(A) 94 (B) 46 (C) 22 (D) 3

Q.76 0, 5, 15, 50, 128
(A) 5 (B) 15 (C) 50 (D) 128

Q.77 9, 63, 5, 35, 1, 8
(A) 63 (B) 5 (C) 35 (D) 8

Q.78 89, 78, 86, 80, 85, 82, 83
(A) 83 (B) 82 (C) 86 (D) 78

Q.79 1,1, 3, 9, 6, 36, 10, 100, 16, 225
(A) 225 (B) 16 (C) 10 (D) 9

Q.80 444, 300, 200, 136, 87, 84, 80
(A) 300 (B) 200 (C) 136 (D) 87

Q.81 8, 15, 31, 61, 123, 247, 491
(A) 247 (B) 491 (C) 121 (D) 61

Q.82 3, 6, 24, 30, 63, 72, 122, 132
(A) 132 (B) 30 (C) 122 (D) 72

Q.83 121, 144, 169, **?** , 225, 256.
(A) 196 (B) 296 (C) 220 (D) 222

Q.84 5, 10, 20, **?** , 80.
(A) 35 (B) 40 (C) 45 (D) 50

Q.85 4, 8, 9, 27, 16, **?** , 25, 125.
(A) 8 (B) 16 (C) 25 (D) 64

Q.86 2, 3, 5, 8, **?** , 17.
(A) 6 (B) 12 (C) 13 (D) 15

Q.87 4, 9, 25, ?, 121, 169.
(A) 36 (B) 49 (C) 64 (D) 81

Q.88 1, 3, 7, 13, 21, ?, 43, 57.
(A) 31 (B) 29 (C) 30 (D) 32

Q.89 5, 3, 10, 8, 17, 15, ?, 24.
(A) 25 (B) 23 (C) 26 (D) 27

Q.90 97, 77, 59, ?, 29, 17.
(A) 34 (B) 39 (C) 37 (D) 43

(Questions 91 – 98)
Instruction : In each of the question Nos. **91** to **98** a number series is given with one term missing shown by question mark (?). This term is one of the four alternatives given under it. Find the correct alternative.

Q.91 1, 2, 3, 15, ?, 56
(A) 31 (B) 40 (C) 37 (D) 45

Q.92 100, 50, $33\frac{1}{3}$, 25, 20, ?,
(A) 15 (B) $16\frac{1}{3}$ (C) $17\frac{2}{3}$ (D) $16\frac{2}{3}$

Q.93 17, 16, 8, _?_, – 83
(A) –1 (B) –8 (C) –19 (D) –26

Q.94 6, 24, 60, 120, _?_.
(A) 180 (B) 195 (C) 210 (D) 225

Q.95 49, 64, 56, 57, 63, _?_, 70, 43
(A) 64 (B) 50 (C) 52 (D) 166

Q.96 4, 13, 31, 67, _?_, 283
(A) 139 (B) 103 (C) 121 (D) 139

Q.97 1, 1, 2, 3, 5, 8, _?_, 21
(A) 11 (B) 12 (C) 13 (D) 14

Q.98 3, 24, 81, ?, 375, 648.
(A) 128 (B) 256 (C) 169 (D) 192

Direction (Q.99 to Q.103) Find the missing term in the series given below.

Q.99 2, 12, 30, ? , 90, 120
(A) 48 (B) 56 (C) 63 (D) 72

Q.100 10, 100, 200, 310, ?
(A) 400 (B) 410 (C) 420 (D) 430

Q.101 0, 5, 2, 4.5, 8, 12.5, ?
(A) 16 (B) 17 (C) 16.5 (D) 18

Q.102 109, 74, 46, 25, 11, ?
(A) 3 (B) 0 (C) 11 (D) 4

Q.103 $\frac{2}{3}, \frac{4}{7}, \frac{?}{?}, \frac{11}{21}, \frac{16}{31}$
(A) $\frac{6}{11}$ (B) $\frac{5}{9}$ (C) $\frac{9}{11}$ (D) $\frac{7}{13}$

Direction (Q. NO. 104 to 107) : In each of the following questions write which term in the sequence replaces the question mark.

Q.104 13 , 23 , 43 , 83 , 163 ,?
(A) 326 (B) 323 (C) 321 (D) 318

Q.105 12 , 15 , 21 , 24 , 30 , 33 ,?,?
(A) 36 , 41 (B) 37 , 42
(C) 38 , 47 (D) 39 , 51

Q.106 16 , 40 , 100 , 250 ,?
(A) 575 (B) 625 (C) 425 (D) 525

Q.107 23 , 29 , 47 , 75 ,?
(A) 87 (B) 93 (C) 110 (D) 117

Instruction: In each of the following question no. 108 to 117, a number series is given with missing of one termthe correct alternative that will continue the same pattern and replace the question mark (?) in the givens

Q.108 2,3,5,9,17
(A) 34 (B) 31 (C) 32 (D) 33

Q.109 2,7,12,17,?,27
(A) 18 (B) 22 (C) 19 (D) 23

Q.110 11,121, 1331,?
(A) 14641 (B) 14411
(C) 14141 (D) 14441

Q.111 656,432, 320, 264, 236,?
(A) 229 (B) 232 (C) 222 (D) 223

Q.112 3,19,97,391,?,2359
(A) 1177 (B) 1084 (C) 1711 (D) 1958

Q.113 2, 7, 24, 77, ?
(A) 1335 (B) 249 (C) 283 (D) 238

Q.114 11, 5, 13,1 0,15,15,17, ?, ?
(A) 5, 11 (B) 20,19
(C) 19,21 (D) 19,20

Q.115 1331,2197,4913,6859,?,24389
(A) 13824 (B) 9261
(C) 12167 (D) 15625

Q.116 97, 86, 99, 88, 101, ?, ?
(A) 90, 103 (B) 88,99
(C) 121,108 (D) 114,103

Q.117 77, 49, 36,18,?
(A) 10 (B) 12 (C) 8 (D) 16

ANSWER KEY

EXERCISE - 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	C	A	D	D	A	D	B	D	D	C	A	D	D	D	B	D	A	B
Que.	21	22	23	24	25															
Ans.	B	C	D	D	C															

EXERCISE - 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	C	A	C	A	A	C	A	C	C	C	C	B	A	C	D	B	B	C
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	B	C	C	B	D	D	C	B	C	A	C	D	B	C	C	B	A	D
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	B	C	C	B	A	B	B	D	A	B	D	A	B	C	B	B	A	C	B
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	A	C	D	C	B	C	C	B	A	B	D	B	A	D	D	D	C	B	D
Que.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	A	C	A	B	D	B	B	A	C	D	A	D	C	C	B	A	C	D	B	D
Que.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117			
Ans.	D	D	D	B	D	B	C	D	B	A	C	A	D	B	C	A	C			

SOLUTIONS

EXERCISE-1

Sol.1 **[C]**

9, 7, 14, 11, 33, 28, 140, 133, 931

2 3 5 7

difference is prime no.

Sol.2 **[A]**

$6 \times 2 + 0 = 12$

$12 \times 2 - 1 = 23$

$23 \times 2 - 2 = 44$

$85 \times 2 - 4 = 166$

Sol.3 **[A]**

92, 23, 27, 9, 12, 6, 8

÷4 +4 :4 +3 ÷3 +3

Sol.4 **[A]**

$1^2 - 1$ 0

$2^2 + 1 = 5$

$3^2 - 1 = 8$

$4^2 + 1 = 17$

$8^2 + 1 = 65$

Sol.5 **[D]**

$999 = 10^3 - 1$

$730 = 9^3 + 1$

$510 = 8^3 - 2$

$345 = 7^3 + 2$

$213 = 63 - 3$

$128 = 5^3 + 3$

Sol.6 **[A]**

$(2 + 5) \times 2 = 14$

$(7 + 13) \times 2 = 2$

$(10 + 4) \times 2 = 28$

Sol.7 **[A]**

$2^2 - 1 = 3$

$+3$ ↷ $3^2 - 2 = 7$

$6^2 - 1 = 35$

$+3$ ↷ $7^2 - 2 = 49$

$10^2 - 1 = 99$

Sol.8 **[D]**

23, 27, 43, 79, 143 —

2^2 4^2 6^2 8^2 10^2

Sol.9 **[B]**

$870 = 29^2 + 29$

$342 = 18^2 + 18$

$132 = 11^2 + 11$

$42 = 6^2 + 3$

$2^0 = 1^2 + 1$

Sol.10 **[D]**

$9 \times 2 + 1 = 18$, $18 \times 2 - 2 - 3 = 35$

$35 \times 2 + 5 = 75$, $75 \times 2 - 7 = 143$

$143 \times 2 + 9 = 295$

Sol.11 **[B]**

2, 20, 110, 380, 650

+18 +90 +270 +270

×5 ×3 ×1

Sol.12 **[C]**

256, 125, 60 ——, 12.5

$\frac{256-6}{2} = 125$

$\frac{256-5}{2} = 60 \Rightarrow \frac{60-4}{2} = 28$

Sol.13 **[A]**

$2 \times 3 = 6$

$3 \times 5 = 15$

$5 \times 7 = 35$

$7 \times 11 = 77$

$19 \times 23 = 437$

Sol.14 **[D]**

9, 9, 14, 34, 124 ——

$9 \times 2 - 4 = 14$

$14 \times 3 - 8 = 34$

$34 \times 4 - 12 = 124$

$124 \times 5 - 16 = 604$

Sol.15 **[D]**

125, 215, 343, 511, 729, 999

↑ ↑ ↑ ↑ ↑ ↑

5^3 $6^3 - 1$ 7^3 $8^3 - 1$ 9^3 $10^3 - 1$

Sol.16 **[D]**

Each term is 4 less than hence the preceding term.

50, 654 is wrong term, it must be $218 \times 3 - 4 = 650$

Sol.17 **[B]**
Each term is 1 mare than twice the preceding term
50, 39 us wrong term

Sol.18 **[D]**
unit place is range term place is increased by from first to tort term.
50, 5686 is wrong term

Sol.19 **[A]**
Logic is : $3^2 + 1, 4^2 - 1, 5^2 - 1, 6^2 - 1, 7^2 + 1$
so 48 is wrong term

Sol.20 **[B]**
2, 11, 25, 46, 109, 221, 445
+7 +14 +28 +56 112 224
×2 ×2 ×2 ×2 ×2
46 is wrong terms

Sol.21 **[B]**
2, 6, 10, 20, 30, 42, 55
+4 +6 +8 +10 +12 +14
10 is wrong term,

Sol.22 **[C]**
$3^1, 3^2, 3^3, 3^4, 3^5$
82 is wrong term,

Sol.23 **[D]**
320, 254, 200, 155, 122, 100, 89
–66 –55 –44 –33 –22 –11

Sol.24 **[D]**
1 + 5 = 6, 3 + 4 = 7, 7 + 1 = 8, 2 + 2 + 3 = 7
3 + 5 + 0 = 8
pattern after adding digital of number 67878 it should be 87876 such type of pattern is called palling drone

Sol.25 **[C]**
5, 10, 40, 80, 320, 550, 2560
×2 ×4 ×2 ×4 ×2 ×4
so, 550 is wrong term

EXERCISE-2

Sol.1 **[B]**
Pattern is $n^2 + 1, n^2 - 1$, where n = 1, 2, 3, 4, 5 ….
So next term is $6^2 - 1 = 35$

Sol.2 **[B]**
it is combination of two series : -
1.27, 125, …..
and
4, 16, 36, …..
pattern is n^2, n^3
So next term is $7^3 = 343$

Sol.3 **[C]**
Logic is $7^3 - 7, 6^3 - 6, 5^3 - 5$, …..
missing term $\Rightarrow 4^3 - 4 = 60$

Sol.4 **[A]**
3, 4, 8, 17, 33, ……
1 4 9 16
difference is $1^2, 2^2, 3^2, 4^2$
next term = 33 $5^2 = 58$

Sol.5 **[C]**
Third term = first term + second terms
missing term = 34 + 55 = 89

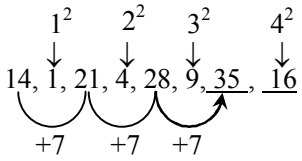
Sol.6 **[D]**
480, 480, 240, 80, 20, 4
÷1 ÷2 ÷3 ÷4 ÷5

Sol.7 **[A]**
1, 1, 2, 2, 3, 4, 4, 8, 5, 16, 6
+1 +1 +1 +1 +1

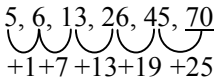
Sol.8 **[C]**
2, 5, 11, 23, 47, ……
2 × 8 + 1 = 5
5 × 2 + 1 = 11
so next term $\Rightarrow$ 47 × 2 + 1 = 95

Sol.9 **[A]**
+11 +11 +11
12, 21, 23, 32, 34, 43, 45, 54
+11 +11 +11

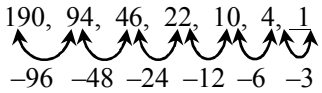
Sol.10 **[C]**



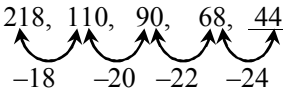
Sol.11 **[C]**



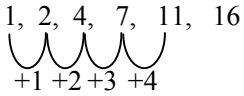
Sol.12 **[C]**



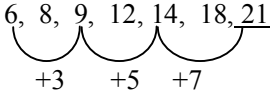
Sol.13 **[C]**



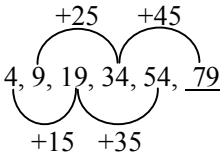
Sol.14 **[B]**



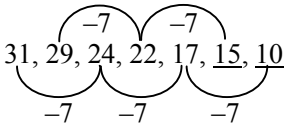
Sol.15 **[A]**



Sol.16 **[C]**



Sol.17 **[D]**



Sol.18 **[B]**

3, 6, 11, 18 ——

logic is $n^2 + 2$, n = 1, 2, 3, ……

missing term = $5^2 + 2 = 27$

Sol.19 **[B]**

Logic is $n^2 - 1$, n = 2, 3, 4, 5, …..

missing term $\Rightarrow 6^2 - 1 = 35$

Sol.20 **[C]**

$4 \times 2 + 2 = 10$

$10 \times 2 + 3 = 23$

$23 \times 2 + 4 = 50$

$50 \times 2 + 5 = 105$

$105 \times 2 + 6 = 216$

Sol.21 **[A]**

Logic is ÷ 3 – 1, ÷3 + 1, …..

$33 \div 3 + 1 = 12$

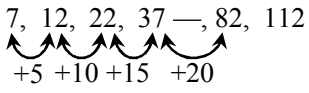
Sol.22 **[D]**

1, 4, 9 ——, 25, 36

all are perfect squares

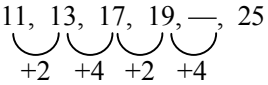
$4^2 = 16$

Sol.23 **[B]**



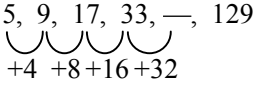
37 + 20 = 57

Sol.24 **[C]**



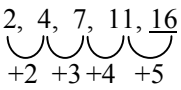
$\Rightarrow$ 19 + 4 = 23

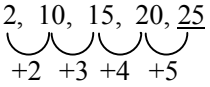
Sol.25 **[C]**



$\Rightarrow$ 33 + 32 = 65

Sol.26 **[B]**

First series = 

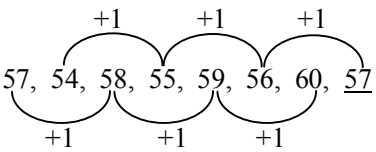
second series = 

Sol.27 **[D]**

Logic is: $n^3 - n$, n = 1, 2, 3, 4, …..

missing term $6^3 - 6 = 210$

Sol.28 **[D]**



Sol.29 **[C]**

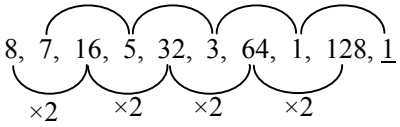
$27 + 2^2 = 31$
$31 + 3^2 = 40$
$40 + 4^2 = 56$
$56 + 5^2 = 81$
$81 + 6^2 = 117$
$117 + 7^2 = 166$

Sol.30 **[B]**

I: 55, 57, 60, 62, 65, 67
+2 +3 +2 +3 +2

II: 168, 120, 80, 48, 24, 8
–48 –40 –32 –24 –16

Sol.31 **[C]**



Sol.32 **[A]**

Logic is ×2 + 1, ×2 – 1, …..
⇒ 131 × 2 – 1 = 261

Sol.33 **[C]**

5, 2, 17, 4, —, 6, 47, 8, 65
+12 +14 +16 +18

⇒ 17 + 14 = 31

Sol.34 **[D]**

1, 2, 4, 8, —, 32
×2 ×2 ×2 ×2 ×2

⇒ 8 × 2 = 16

Sol.35 **[B]**

2, 3, 10, 15, 26, —
1^2+1 2^2-1 3^2+1 4^2-1 5^2+1 6^2-1

⇒ 36 – 1 = 35

Sol.36 **[C]**

It is combination of two series :
I. 2, 6, 12, —
II. 30, 20, 12,
pattern in I is $1^2 + 1$, $2^2 + 2$, $3^2 + 3$, ….
missing term = $4^2 + 4 = 20$

Sol.37 **[A]**

$1 \times 1 \times 6 = 6$
$2 \times 2 \times 5 = 20$
$3 \times 3 \times 4 = 36$
$4 \times 4 \times 3 = 48$
$5 \times 5 \times 2 = 50$
$6 \times 6 \times 1 = 36$
$7 \times 7 \times 0 = 0$

Sol.38 **[B]**

Logic is × 2 + 1, × 2 – 2, × 2 + 3,
missing term ⇒ 114 × 2 + 5 = 233

Sol.39 **[A]**

25, 49, 89, 145, 217, —
+24 +40 +56 +72 +88
+16 +16 +6/6 +16

217 + 88 = 305

Sol.40 **[D]**

It is combination of three series :
+3 +5 +7
I: 0, 3, 8, ?
II: 2, 3, 4, 5
III : 2, 5, 10, 17
messing term : 8 + 7 = 15

Sol.41 **[C]**

Logic is: $n^2 – n$, n = 1, 2, 3, 4,
$1^1 – 1 = 0$,
$2^2 – 2 = 2$
$5^5 – 5 = 3125 – 5 = 3120$

Sol.42 **[B]**

Logic is $n^3 – n$, n = 2, 3, 4, 5
so missing term = $6^3 – 6 = 210$

Sol.43 **[C]**

Logic is: $3^n – 1$, $3^n + 1$, $3^n – 1$, $3^n + 1$, …..
so missing term ⇒ $3^4 + 1 = 82$

Sol.44 **[C]**

5, 10, 17, 26, 37, 50, 65
+5 +7 +9 +11 +13 +15

Sol.45 **[B]**

Logic is $n^3 – 2$, n = 2, 3, 4, 5,…..
missing term ⇒ $6^3 – 2 = 214$

Sol.46 **[A]**

5, 3, 10, 8, 17, 15, —, 24

+5 +7 +9

missing term $\Rightarrow 17 + 9 = 26$

Sol.47 **[B]**

Logic is $n^2 - n$, n = 2, 3, 4, 5,….

missing term $\Rightarrow 7^2 - 7 = 42$

Sol.48 **[B]**

12, 22, 69, 272, 1365, 8184

×2–2 ×3+3 ×4–4 ×5+5 ×6–6

Sol.49 **[D]**

I : 65, 64, 63

II: 48, 49, 50

+1 +1

Sol.50 **[A]**

Logic is $n^2 - 2$, n = 3, 5, 7, 9, ……

missing term = $7^2 - 2 = 47$

Sol.51 **[B]**

16, 8, 12, —, 105

×0.5 ×1.5 ×2.5 ×3.5

missing number $\Rightarrow 12 \times 2.5 = 30$

Sol.52 **[D]**

–1 –1 –1 –1 –1

748, 737, 716, 685, 644, 593

–1 –2 –3 –4 –5

Sol.53 **[A]**

Logic is n^3, n = 2, 3, 4, 5, …..

missing number $5^3 = 125$

Sol.54 **[B]**

5, 11, 19, 29, 41

↑ ↑ ↑ ↑ ↑

2^2+1 3^2+2 4^2+3 5^2+4 6^2+5

Sol.55 **[C]**

120, 60, 24, 6, 0

↑ ↑ ↑ ↑ ↑

5^3-5 4^3-4 3^3-3 2^3-2 1^3-1

Sol.56 **[B]**

Logic is divide by 9

missing term $\frac{1}{9} \div 9 = \frac{1}{81}$

Sol.57 **[B]**

It concept of two series

I: 3, 9, 27, 81, 243

×3 ×3 ×3 ×3

II: 7, 21, 66, 189,

×3 ×3

$21 \times 3 = 63$

wrong term = 66

Sol.58 **[A]**

Logic is +7, +6, +5, +4, …… +1

53 is wrong term

Sol.59 **[C]**

I: 0, 3, 6, 9, 12

+3 +3 +3 +3

II: 2, 6, 20, 54

×3 ×3 ×3

So, 20 is wrong term

Sol.60 **[B]**

logic is $0^3 + 0$, $1^3 = 1$, $2^3 + 2^3 + 2$, $3^3 + 3$, …..

wrong term is 36

Sol.61 **[B]**

9, 54, 44, 264, 754, 1520, 1514

×6 –10 ×6 –10 ×6

$\Rightarrow 254 \times 6 = 1524$

wrong term is 1520

Sol.62 **[A]**

Sequence is :

3^2+1, 4^2-1, 5^2+1, 6^2-1, 7^2+1, 8^2-1,…….

so, 48 is wrong term and replaced by

Sol.63 **[C]**

The sequence is :

1^3+2, 2^3+2, 3^3+2, 4^3+2, ……..

so, 30 is wrong term

Sol.64 **[D]**

Difference between terms are : -

$+1^2$, $+3^2$, $+5^2$, $+7^2$, $+9^2$

so, 9 is wrong term

Sol.65 **[C]**
Sequence in the given relies terms
+10, ×2, +10, ×2, + 10, ×2, …….
so, 158 is wrong term

Sol.66 **[B]**
2, 6, 10, 20, 30, 42, 56
+4 +6 +8 +10 +12 +14
10 is wrong term

Sol.67 **[C]**
7 + 9 = 16, 9 + 16 = 25, 16 + 25 = 41
25 + 41 = 66
so 68 is wrong term

Sol.68 **[C]**
$3^1, 3^2, 3^3, 3^4, 3^5$
82 is wrong term

Sol.69 **[B]**
5 × 2 – 1 = 9
9 × 2 – 1 = 17
17 × 2 – = 33
so, 35 is wrong term

Sol.70 **[A]**
1 + 5 = 6, 6 + 5 =11, 11 + 6 = 17
17 + 11 = 28
so, 27 is wrong term

Sol.71 **[B]**
3, 6, 11, 18, 28, 38, 51, 61
+3 +5 +7 +9
28, is wrong term

Sol.72 **[D]**
320, 254, 200, 155, 122, 100, 89
+66 +55 +44 +33 +22 +11
200 is wrong term

Sol.73 **[D]**
I: 6, 9, 14, 22, 30
+3 +5 +7 +9
II: 8, 12, 18, 26
+4 +6 +8
22 is wrong term

Sol.74 **[A]**
I : 3, 9, 27, 81, 243
II: 7, 28, 84, 448
+4 +4 +4
28 × 4 = 112
so, 84 is wrong term

Sol.75 **[D]**
3, 10, 22, 46, 94, 190
×2 +2 ×2 +2 ×2 +2 ×2 +2 ×2 +2 ×2 +2
190 is wrong term

Sol.76 **[D]**
0, 5, 15, 50, 128
×1 +5 ×2 +5 ×3 +5 ×4 +5
128 is wrong term

Sol.77 **[D]**
9, 63, 5, 35, 1, 8
I: 9, 5, 1
63, 35, 8
8^2-1 6^2-1 4^2-1
8 is wrong term

Sol.78 **[C]**
I: 78, 80, 82
II: 89, 86, 85, 83
86 is wrong term

Sol.79 **[B]**
I: 1, 3, 6, 10, 16
+2 +3 +4 +5

Sol.80 **[D]**
440, 300, 200, 136, 87, 84, 80
144 100 64 4
12^2 10^2 8^2 2^2

Sol.81 **[A]**
8, 15, 31, 61, 123, 247, 491
×2–1 ×2+1 ×2–1 ×2+1 ×2–1 ×2+1

Sol.82 **[A]**
3 × 1 = 3, 3 × 2 = 6
6 × 4 = 24, 6 × 5 = 30
9 × 7 = 63, 9 × 8 = 92
12 × 10 = 120, 12 × 11 = 132

Sol.83 **[A]**
$11^2,\ 12^2,\ 13^2,\ 14^2,\ 15^2,\ 16^2$
missing term = 196

Sol.84 **[B]**
5, 10, 20, —, 80
×2 ×2 ×2 ×2
$20 \times 2 = 40$

Sol.85 **[D]**
$2^2,\ 2^3,\ 3^2,\ 3^3,\ 4^2,\ 4^3,\ 5^2,\ 5^3,$ …..
$4^3 = 64$

Sol.86 **[B]**
2, 3, 5, 8, —, 17
+1 +2 +3 +4 +5

Sol.87 **[B]**
Square of prime number
$7^2 = 49$

Sol.88 **[A]**
$1^2 - 0 = 1,\ 2^2 - 1 = 3,\ 3^2 - 2 = 7,\ 4^2 - 3 = 13,$….
$5^2 - 4 = 21,\ 6^2 - 5 = 31$

Sol.89 **[C]**
I: 5, 10, 17, —
+5 +7 +9
II: 3, 8, 15, 24
+5 +7 +9
17 + 9 = 26

Sol.90 **[D]**
Logic is: $10^2 - 3,\ 9^2 - 4,\ 8^2 - 5,\ 7^2 - 6,$ ……
missing term = $7^2 - 6 = 43$

Sol.91 **[A]**
1, 2, 6, 15, —, 56
$+1^2$ $+2^2$ $+3^2$ $+3^2$ $+5^2$
15 + 16 = 31
places do correction in question, in sequence replace 3 by 6

Sol.92 **[D]**

$100,\ 50,\ 33\frac{1}{3},\ 25,\ 20,$ —

$100 \div 1 = 100,\ 100 \div 2 = 50,\ 100 \div 3 = 33\frac{1}{2}$

So, …… $100 \div 6 = 16\frac{2}{3}$

Sol.93 **[C]**
17, 16, 8, —, –83
-1^3 -2^3 -3^3 -4^3
so, $8 - 3^3 = -19$

Sol.94 **[B]**
$n^3 - n,$ n = 2, 3, 4,…..
missing term $6^3 - 6 = 210$

Sol.95 **[B]**
I: 49, 56, 63, 70
+7 +7 +7
II: 64, 57, —, 43
–7 –7 –7
57 – 7 = 50

Sol.96 **[A]**
Logic is ×5 + 5
missing term is $67 \times 2 + 5 = 139$

Sol.97 **[C]**
1 + 1 = 2
2 + 1 = 3
3 + 5 = 8
5 + 8 = 13
missing term = 13

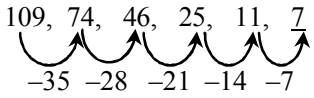
Sol.98 **[D]**
$1^3 \times 3 = 3$
$2^3 \times 3 = 24$
$3^3 \times 3 = 81$
$4^3 \times 3 = 192$

Sol.99 **[B]**
$2 \times 1 = 2$
$3 \times 4 = 12$
$5 \times 6 = 30$
$7 \times 8 = 56$
$9 \times 10 = 90$
$11 \times 12 = 132$

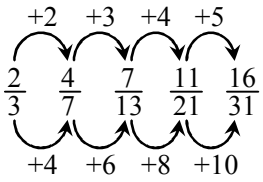
Sol.100 **[D]**
10, 100, 200, 310, —
+90 +100 +110 +120
$\Rightarrow 310 + 120 = 430$

Sol.101 **[D]**
0, 0.5, 2, 4.5, 8, 12.5, —
+0.5 +1.5 +3.5 +4.5 +5.5
12.5 + 5.5 = 18

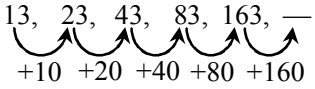
Sol.102 [D]



Sol.103 [D]



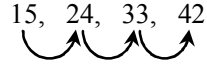
Sol.104 [C]



Sol.105 [D]

I: 12, 21, 30, 39

II: 15, 24, 33, 42



39, 42

do correction in option

Sol.106 [B]

$\frac{16\times5}{2}=40$

$\frac{40\times5}{2}=100$

$\frac{100\times5}{2}=250$

$\frac{250\times5}{2}=625$

Sol.107 [C]

$23 + 2 \times 3 = 29$

$29 + 2 \times 9 = 47$

$47 + 4 \times 7 = 75$

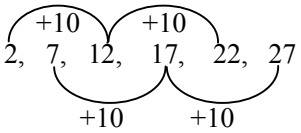
$75 + 7 \times 5 = 110$

Sol.108 [D]

$\times 2^{-1}$ is logic

$17 \times 2 - 1 = 33$

Sol.109 [B]



Sol.110 [A]

$11^1, 11^2, 11^3, 11^4$

$11^4 = 14641$

Sol.111 [C]

$\text{Logic is next term} = \frac{\text{revious term}}{2} + 104$

$\frac{656}{2} + 104 = 432$

$\frac{432}{2} + 104 = 320$

$\frac{236}{2} + 104 = 232$

Sol.112 [A]

$3 \times 6 + 1 = 19$

$19 \times 5 + 2 = 97$

$97 \times 4 + 3 = 391$

$391 \times 3 + 4 = 1177$

Sol.113 [D]

$2 \times 3 = 1 = 7$

$7 \times 2 + 3 = 21$

$77 \times 3 + 238$

Sol.114 [B]

I : 11, 13, 15, 17, 19

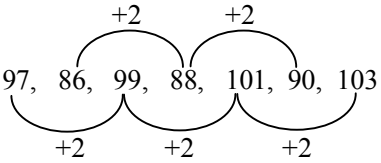
II: 5, 10, 15, 20

Sol.115 [C]

$11^3, 13^3, 17^3, 19^3, 23^3, 29^3$

$23^3 = 12167$

Sol.116 [A]



Sol.117 [C]

$7 \times 7 = 49$

$4 \times 9 = 36$

$3 \times 6 = 18$

$1 \times 8 = 8$

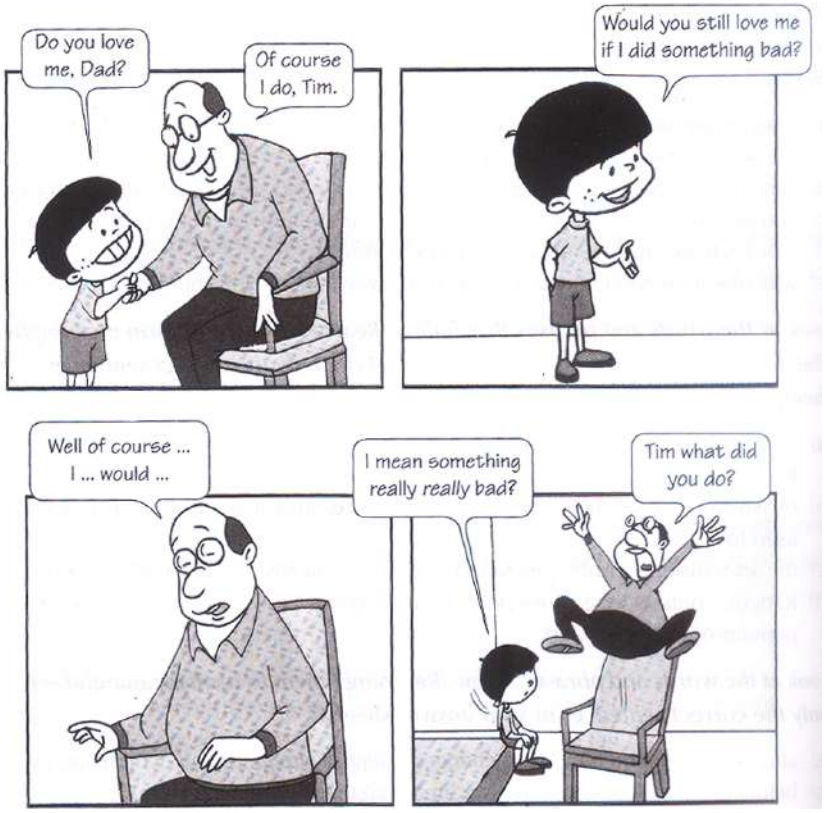
ENGLISH

Chapter 4

INTEGRATED GRAMMAR

[A] Dialogue Completion

Q.1 Read the comic strip and complete the passage that follows. Write the correct answers in your answer sheets against the correct blank numbers. Do not copy the whole sentence.

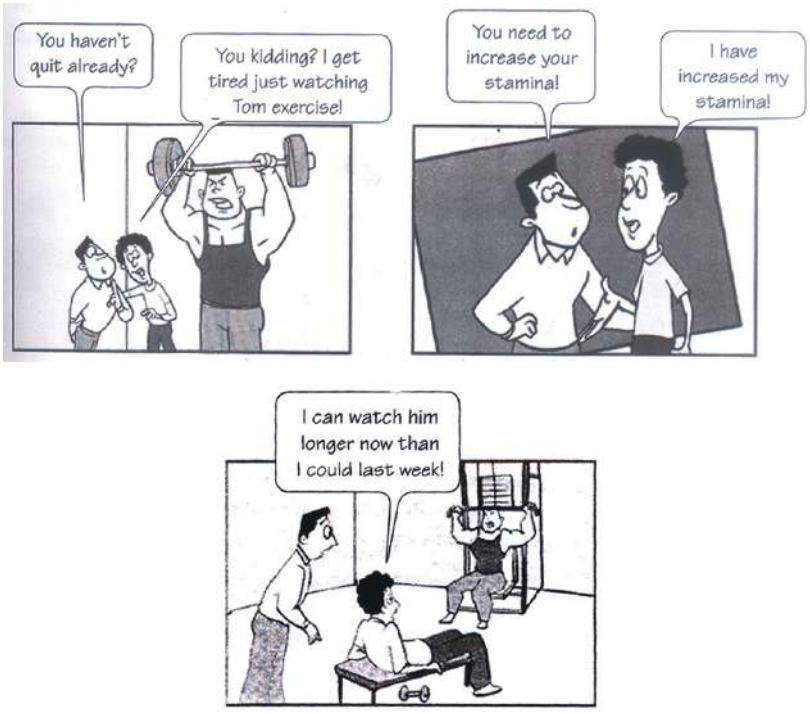


One evening Tim asked his father (a)_____. His father replied that (b)_____. Then Tim wanted to know (c)_____. His father once again replied (d)_____. But when Tim kept on insisting his father got suspicious and shouted at asking him (e)_____.

Q.2 Read the comic strip and complete the passage that follows. Write the correct answers in your answer sheets against the correct blank numbers. Do not copy the whole sentence.

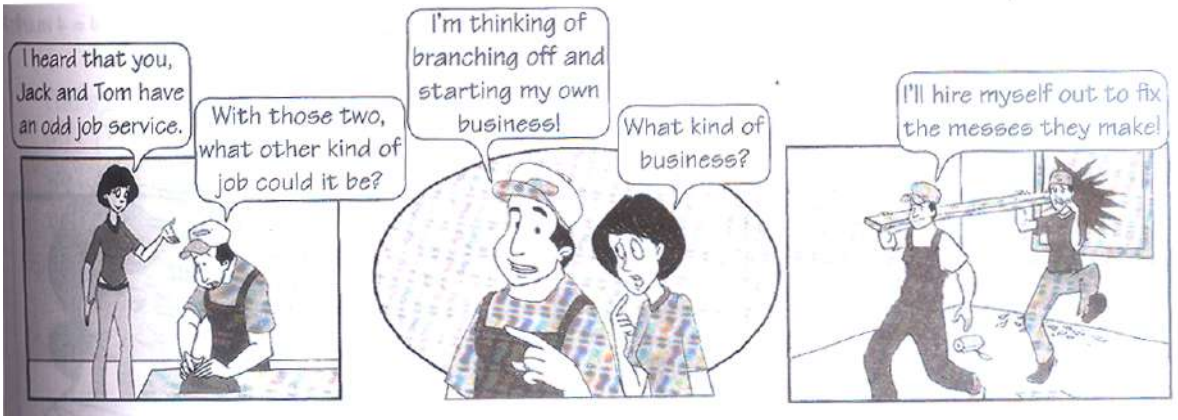
Rohan went up to a policeman and informed him (a)_____. The policeman wanted to know (b)_____. Rohan replied (c)_____at that time. Then the policeman wanted to know at what time he had been there and for how long. Rohan said (d)_____from 7 p.m. to 9 p.m. Finally the policeman said that (e)_____be tough.

Q.3 Read the comic strip and complete the passage that follows. Write the correct answer in your answer sheets against the correct blank numbers. Do not copy the whole sentence.



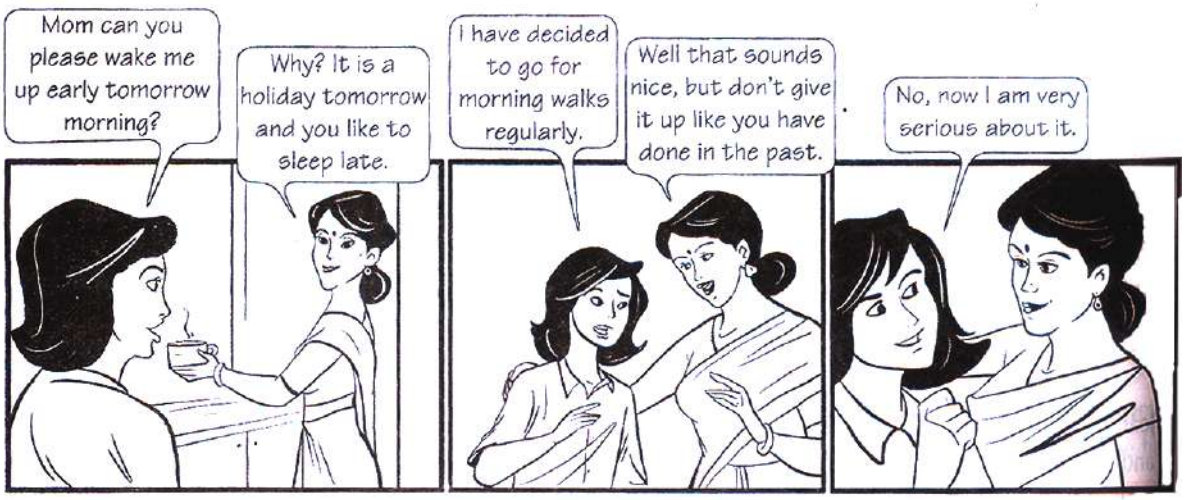
Will was surprised to see Danny working out and so he wanted to know (a)_____. Dann replied that he (b)_____. Will said that he (c)_____.To this Danny replied that (d)_____for he could now (e)_____.

Q.4 Read the comic strip and complete the passage that follows. Write the correct answers in your answer sheets against the correct blank numbers. Do not copy the whole sentence.



Jean told Stuart that she had heard that (a)_____. He replied that (b)_____ and he was thinking of (c)_____.Then Jean wanted to know (d)_____. Stuart replied that (e)_____.

Q.5 Read the comic strip and complete the passage that follows. Write the correct answer in your answer sheets against the correct blank numbers. Do not copy the whole sentence.



Seeta asked her mother (a)_____. Her mother was surprised and wanted to know why (b)_____sleep late. She replied that she had decided to go for morning walks regularly. The mother was pleased but asked her not (c)_____. Seeta replied that now (d)_____.

Q.6 Read the comic strip and complete the passage that follows. Write the correct answer in your answer sheets against the correct blank numbers. Do not copy the whole sentence.

Eddie's friend asked him (a)_____. Eddie replied that Barbie's team needed it. She asked him (b)_____.Eddie replied that (c)_____.Then his friend said that in that case (d)_____.Eddie replied that (e)_____leaving his friend looking all upset and confused with his way of thinking.

Q.7 Read the comic strip and complete the passage that follows. Write the correct answers in your answer sheets against the correct blank numbers. Do not copy the whole sentence.

The librarian asked the student (a)_____library. The student exclaimed that he (b)_____friend. The librarian replied that the library (c)_____and all (d)_____ read books. The student protested that (e)_____. Then the librarian remarked that it was no wonder that he did not get good marks in English.

Q.8 Read the conversation given below and complete the passage that follows. Write the answers against the correct blank numbers. Do not copy the whole sentences.

Bipasha : Why are you not participating in the inter-school singing competition ? You are the best singer of the school.

Mallika : I will participate only when I get an opportunity to perform individually.

Bipasha : Mallika, it is not a good thing to look down upon others.

Bipasha asked Mallika (a) _____ participating in the inter-school singing competition. She reminded her (b) _____ of the school. Mallika proudly replied that (c) _____ an opportunity to perform individually. Seeing her attitude Bipasha told her (d) _____ to look down upon others.

(a) (i) why was she not
(ii) why she was not
(iii) why did she not
(iv) why she didn't

(b) (i) she was the best singer
(ii) that she is the best singer
(iii) that she is a good singer
(iv) that she was the best singer

(c) (i) she would participate only if she got
(ii) she would participate only if she gets
(iii) she would participate only when she got
(iv) she would be participating only when she got

(d) (i) that it was not a good thing
(ii) it was not a good thing
(iii) it is not a good thing
(iv) that it is not a good thing

[B] Editing

Q.1 The following passage has not been edited. There is one error in each of the lines. Write incorrect word and the correction in your answer sheets.

		Incorrect	Correct
Theatre first develop in Greece as a part	(a)	_____	_____
of religious observe. The stage was simply	(b)	_____	_____
a circle of turf on who the worshippers danced,	(c)	_____	_____
around a altar of Dionysus. The spot was usually	(d)	_____	_____
at the foot of a hill so that the spectator on the	(e)	_____	_____
slope would watch the dance. This started the	(f)	_____	_____
tradition of Greek theatres-semicircles of seats build	(g)	_____	_____
into a hillside. A theatre built on Athens in 500 BC	(h)	_____	_____
had a circular place, called the orchestra, when the	(i)	_____	_____
performance is given.	(j)	_____	_____

Q.2 The following passage has not been edited. There is one error in each of the lines. Write the incorrect word and the correction in your answer sheets.

		Incorrect	Correct
The body could be consider a permanent furnace.	(a)	_____	_____
The food we take in is 'fuel', who the body 'burns up'.	(b)	_____	_____
In these process, about 2,500 calories are being used	(c)	_____	_____
every day in the body. Its heat enough to bring 23 litres	(d)	_____	_____
of water to the boiling point! What happened to all	(e)	_____	_____
this heat in the body? If there were not temperature	(f)	_____	_____
controls in the body, we could certainly think of ourself	(g)	_____	_____
as 'hot stuff'. But we all know that the heat			
of the body don't go up (unless we're sick). We know	(h)	_____	_____
that our body heat remains at a average temperature	(i)	_____	_____
of 37 degree centigrade. Perspiration are one of the	(j)	_____	_____
ways to keep our body 'furnace' in a nice normal	(k)	_____	_____
temperature. Actually, our body temperatures is	(l)	_____	_____
controlled by a centre in the brain known as the			
temperature centre.			

Q.3 The following passage has not been edited. There is one error in each of the lines. Writ the incorrect word and the correction in your answer sheets.

		Incorrect	Correct
At Haridwar begin the milking of the	(a)	_____	_____
Ganges. Canal take off into different	(b)	_____	_____
directions, left a trickle in the main stream.	(c)	_____	_____
One canal is use to produce electric power,	(d)	_____	_____
another goes to irrigation thirsty flat lands	(e)	_____	_____
of UP. A water of these canals or that	(f)	_____	_____
running on taps is not sacred;	(g)	_____	_____
those privilege is reserved.	(h)	_____	_____

Q.4 The following passage has not been edited. There is one error in each of the lines. Write th incorrect word and the correction in your answer sheets.

		Incorrect	Correct
"0 sweet Portia, here are a few from the most	(a)	_____	_____
unpleasant words that were ever written. For enable me	(b)	_____	_____
to come here by you, my dear friend Antonio had borrowed	(c)	_____	_____
money for me in credit. He had borrowed the money	(d)	_____	_____
by Shylock, a Jew, who made him sign a peculiar bond.	(e)	_____	_____
It said that if Antonio could not return the money he			
had loaned from a particular date, he would have	(f)	_____	_____
to allow Shylock to cut off an exact pound from his flesh	(g)	_____	_____
near his heart, " said Bassanio, ready to collapse at worry.	(h)	_____	_____

Q.5 The following passage has not been edited. There is one error in each of the lines. Write the incorrect word and the correction in your answer sheets.

		Incorrect	Correct
The house on which we were to live	(a)	_____	_____
was in one end of the village.	(b)	_____	_____
It was hiding behind a screen of	(c)	_____	_____
mango and orange tree and bushes	(d)	_____	_____
of hibiscus full from enormous	(e)	_____	_____
scarlet flowers. The house were adequate	(f)	_____	_____
without be luxurious. We had just finished	(g)	_____	_____
unpacking when they were greeted by the	(h)	_____	_____
housekeeper which name was Paula.	(i)	_____	_____

Q.6 The following passage has not been edited. There is one error in each of the lines. Write the incorrect word and the correction in your answer sheets.

	Incorrect	Correct
Industrial and urbanization have resulted	(a) _____	_____
in profound deteriorating of India's air quality.	(b) _____	_____
Of the 3 million premature death in the world that	(c)_____	_____
occur each year because to outdoor and indoor air	(d) _____	_____
pollution, the highest number have been assessed to	(e) _____	_____
occur in India. According to a World Health	(f) _____	_____
Organization, the capital city of New Delhi is all of	(g) _____	_____
the top ten more polluted cities in the world.	(h) _____	_____

Q.7 The following passage has not been edited. There is one error in each of the lines. Write the incorrect word and the correction in your answer sheets.

	Incorrect	Correct
Dance is an art form is	(a) _____	_____
well known, for dance as a	(b) _____	_____
therapy is not known in many.	(c) _____	_____
Dance therapy involves a synthesis	(d) _____	_____
of the grace and vigour on Indian	(e) _____	_____
classical and folk dance movements into a innovative	(f) _____	_____
holistic therapy. It brings over the	(g) _____	_____
inner feelings for the participants	(h) _____	_____
and can help them with develop a healthy personality.	(i) _____	_____

Q.8 The following passage has not been edited. There is one error in each of the lines. Write the incorrect word and the correction in your answer sheets.

	Incorrect	Correct
A curious thing about the developed of	(a) _____	_____
a motion picture is that the first groups of	(b) _____	_____
people who made it possible wasn't interested	(c) _____	_____
in movies at all! The first inventions were make by	(d) _____	_____
men who wanted to study the movement of animals.	(e) _____	_____
Even Thomas Edison, which perfected a device	(f) _____	_____
called 'kinetoscope' in 1893, think of it	(g) _____	_____
only as a curiosity. But there are many other people	(h) _____	_____
who saw great possibility for entertainment	(i) _____	_____
in these inventions and they begin to make movies.	(j) _____	_____

[C] Omission

Direction : In the passages given below, one word has been omitted in each line. Write the missing word along with the word that comes before and the word that comes after it in your answer sheets. Ensure that the word that forms your answer is underlined.

Q.1

		Before	missing	after
In the evening when we reached home				
of us were too tired to even think.	(a)	_____	_____	_____
All decided to have tea and snacks.	(b)	_____	_____	_____
Being eldest I had to prepare everything.	(c)	_____	_____	_____
I opened the refrigerator to find there wasn't milk	(d)	_____	_____	_____
to prepare ten cups of tea and very butter to make	(e)	_____	_____	_____
sandwiches. Somehow I managed to prepare tea.	(f)	_____	_____	_____
Everyone enjoyed tea and the snacks that	(g)	_____	_____	_____
I served but there wasn't tea left for me.	(h)	_____	_____	_____

Q.2

		Before	missing	after
Macbeth not really like to play fool.	(a)	_____	_____	_____
But the witches promised him greatness,	(b)	_____	_____	_____
and already two of their predictions had proved true.	(c)	_____	_____	_____
Now his mind to be obsessed with the intense	(d)	_____	_____	_____
desire for the third prediction to come true. It only	(e)	_____	_____	_____
human to want this greatness. As the seeds of ambition	(f)	_____	_____	_____
growing his mind to think dark thoughts about getting	(g)	_____	_____	_____
rid of Duncan and his sons. Being a king a very attractive	(h)	_____	_____	_____
proposition indeed.				

Q.3

		Before	missing	after
"This the noblest Roman of them all,"	(a)	_____	_____	_____
he said. "Except for him, all the conspirators what	(b)	_____	_____	_____
they did in envy of great Caesar." Brutus the	(c)	_____	_____	_____
only one who had this terrible deed for what he	(d)	_____	_____	_____
honestly to be for the good of Rome. Antony	(e)	_____	_____	_____
recognised this. He his last tribute, 'His	(f)	_____	_____	_____
life gentle," he declared, "and the elements were	(g)	_____	_____	_____
so mixed in him that Nature might stand up and to	(h)	_____	_____	_____
all the world, "This was a man!"				

Q.4 Finally, an idea them. For the journey, Rosalind would herself as a man, and arm herself with spear and axe. She Celia that she would herself Ganymede. Celia to be a maid, and changed her name to Aliena. Now they were for their journey to the Forest of Arden.

	Before	missing	after
(a)	_____	_____	_____
(b)	_____	_____	_____
(c)	_____	_____	_____
(d)	_____	_____	_____
(e)	_____	_____	_____
(f)	_____	_____	_____

Q.5 At Dover all his bags opened by the customs man. Then he to his car. The customs officer who looking at the car was a man he well, a man about his own age. Selby him a big smile, and answered the usual questions. Everything well, just as it always, the smile, the joke, the easy manner, and the promise of a free seat, if the officer would him for it

	Before	missing	after
(a)	_____	_____	_____
(b)	_____	_____	_____
(c)	_____	_____	_____
(d)	_____	_____	_____
(e)	_____	_____	_____
(f)	_____	_____	_____
(g)	_____	_____	_____
(h)	_____	_____	_____

Q.6 The idea of police system to protect a city originated London. In 1737, a law was passed creating a police system 68 men. But as the city grew poverty increased, looting and rioting were soon out control in London. In 1829, Sir Robert Peel formed London Metropolitan Police, with headquarters in Scotland Yard. This new force Peel had created was larger, better trained, and more highly disciplined any other police force ever been.

	Before	missing	after
(a)	_____	_____	_____
(b)	_____	_____	_____
(c)	_____	_____	_____
(d)	_____	_____	_____
(e)	_____	_____	_____
(f)	_____	_____	_____
(g)	_____	_____	_____
(h)	_____	_____	_____
(i)	_____	_____	_____
(j)	_____	_____	_____

Q.7 From humble beginnings a rural French Canadian hometown, Celine Dion risen to international superstardom a shooting star. Born in Charlemagne, Celine the youngest of 14 children of highly musical family. Her parents, musicians, operated a small club, and weekends, the entire family performed and entertained local population. From the tender age of 5, Celine sang her siblings. At the age of twelve, together with her mother and her brothers Celine composed a French song which altered the course of her life.

	Before	missing	after
(a)	_____	_____	_____
(b)	_____	_____	_____
(c)	_____	_____	_____
(d)	_____	_____	_____
(e)	_____	_____	_____
(f)	_____	_____	_____
(g)	_____	_____	_____
(h)	_____	_____	_____
(i)	_____	_____	_____
(j)	_____	_____	_____

		Before	missing	after
Q.8	Basketball is a sport can be played	(a) _____	_____	_____
	by men, women children. It is usually	(b) _____	_____	_____
	played on a wooden court in gymnasium.	(c) _____	_____	_____
	There a backboard and a hoop, called basket,	(d) _____	_____	_____
	in the centre of end of the court. The rim of the	(e) _____	_____	_____
	basket is 3 m the floor. The diameter of the rim	(f) _____	_____	_____
	is 46 cm. A basketball is round like a soccer ball, it is	(g) _____	_____	_____
	bigger. Two teams of five people play each other. A team	(h) _____	_____	_____
	wins a game scoring more points than the other team.	(i) _____	_____	_____
	Players score points by throwing the ball the basket.	(j) _____	_____	_____

		Before	missing	after
Q.9	Russia and the U.S. decided stop being enemies.	(a) _____	_____	_____
	They longer wanted to blow each other up with bombs.	(b) _____	_____	_____
	Each side stopped making bombs. America no	(c) _____	_____	_____
	longer tests nuclear weapons. There no longer any	(d) _____	_____	_____
	money to support making nuclear weapons, the	(e) _____	_____	_____
	piles of nuclear weapons still around. They need	(f) _____	_____	_____
	to be watched over to keep safe. The	(g) _____	_____	_____
	scientists made the bombs still have a lot	(h) _____	_____	_____
	of their research information. They not	(i) _____	_____	_____
	want to get rid of it. They decided to save for	(j) _____	_____	_____
	the others to use and learn from.			

		Before	missing	after
Q.10	Today's business scene even non-commercial organizations	(a) _____	_____	_____
	needs superior administrative skills, particularly managing	(b) _____	_____	_____
	people and systems - which require combination of this	(c) _____	_____	_____
	consistent performance along the nimbleness of mind	(d) _____	_____	_____
	and body respond to minor crises. No wonder women are	(e) _____	_____	_____
	better equipped here well. The strange thing is that	(f) _____	_____	_____
	this has not been recognized and given due credit.			

[D] JUMBLED WORDS

Direction : Rearrange the words and phrases below to form meaningful sentences. Write the corrected sentences in your answer sheet against the correct blank number. The first one has been done as an example.

Q.1 is one / summer visitors / the / of the / swallow / best known

The swallow is one of the best known summer visitors.

(a) a sheen / and wings / forked tail / on / it has / the back / a long

(b) beneath it / at / with / dark red / its throat / a / it has / blue band.

(c) often / open space / swallows / over / seen skimming / a pond / or / are / an

(d) sometimes / a busy / they may / down / be found / city street / hunting insects.

Q.2 retailer / ice - cream / to tell / the Swad / has a story.

The swad ice-cream retailer has a story to tell.

(a) is a / ice-cream / that / tough business / selling / he feels.

(b) now a days / in the market / there are just / selling / too many / ice - creams / of / brands.

(c) there were / varieties / just / two or three / different / a decade ago /from / to choose.

(d) try / some people / a new brand / who / come back / again and again / them / for.

(e) it's the King cones / hot cakes / sell like / that.

(f) some brands / customers / favoured by / there are / which are not / however.

Q.3 (a) pollution / efforts / by all / concerted / be made / should / to curb / environmental.

(b) fuels / burning / is / fossil / discouraged / to be / of.

(c) industries / residential / shifted / polluting / areas / the / far off / must be / from.

(d) the roads / more trees / vacant fields / and / along / in / be grown / should.

(e) impossible / but / certainly not/ task / difficult / it / a / will be/ though.

Q.4 (a) to / with / good / has / what / do / it / being ?

(b) work / you / rewarded / your / good / be / for / will.

(c) medals / show / the / your / principal / to.

(d) have / foolishly / how / behaved / I !

Q.5 (a) desire / yield / I / how / your / to / can ?

(b) the / difficult / were / how / questions !

(c) law / discovered / Newton / of / had / gravitation / the.

(d) Mix / boys / up / bad / don't / the / with.

Q.6 (a) concentration / training / is / upon / autogenic / based / passive.

(b) disease alleviating / the status / it / capabilities / has been/ a therapy / given / by its / of.

(c) training / stress / for / used / reducing / is / autogenic / effectively.

(d) adverse / exercise / are felt / some / discontinued / if / the / be / must / side effects.

(e) people / disorders / recommended / with / it / severe / is / for / mental / not.

ANSWER KEY

INTEGRATED GRAMMAR

[A] Dialogue Completion

1. (a) if he loved him

(b) he certainly did

(c) if he would still love him if he did something bad

(d) he certainly would

(e) what he had done

2. (a) that he had lost his purse.

(b) where he was when he lost the suitcase.

(c) that he had been on the beach.

(d) that he had been there.

(e) he would try to find his suitcase, but that was going to be tough.

3. (a) if he hadn't quit already.

(b) got tired just watching Tom exercise.

(c) needed to increase his stamina.

(d) he had increased his stamina.

(e) watch him longer than he could the previous week.

4. (a) he, Jack and Tom had an odd job service

(b) he was thinking of branching off

(c) starting his own business

(d) what kind of business

(e) he would hire himself out to fix the messes they made

5. (a) if she could wake her up early next morning

(b) as it was a holiday the next day and she liked to sleep late

(c) to give it up like she had done in the past.

(d) she was very serious about that

6. (a) if he was going to walk clear across town to lend someone his baseball glove

(b) why they didn't ask him to play

(c) they didn't need him ; they needed his glove

(d) Barbie would come and get it herself

(e) he was just trying to be nice

7. (a) not to talk loudly in the

(b) wanted to discuss an answer with his

(c) was not a place for discussion

(d) they could do was

(e) he didn't like reading books

8. (i) - (b) (ii) - (d) (iii) - (c) (iv) - (a)

[B] Editing

1.	(a) develop	developed
	(b) observe	observence
	(c) who	which
	(d) a	an
	(e) spectator	spectators
	(f) would	could
	(g) build	built
	(h) A	The
	(i) when	where
	(j) is	was

2.

(a) consider	considered
(b) who	which
(c) these	this
(d) heat enough	hot enough
(e) happened	happens
(f) not	no
(g) ourself	ourselves
(h) don't	doesn't
(i) a	an
(j) are	is
(k) in	at
(l) temperatures	temperature

3.

(a) begin	began
(b) take	takes
(c) left	leaving
(d) use	used
(e) irrigation	irrigare
(f) A	The
(g) on	from
(h) those	this

4.

(a) from – of	(b) For – To
(c) by – to	(d) in – on
(e) by – from	(f) from – by
(g) from – of	(h) at – with

5.

Incorrect	**Correct**
(a) in	at
(b) hiding	hidden
(c) tree	trees
(d) from	of
(e) were	was
(f) be	being
(g) they	we
(h) which	whose

6.

(a) Industrial	Industrialisation
(b) deteriorating	deterioration
(c) death	deaths
(d) to	of
(e) have	has
(f) a	the
(g) all	one
(h) more	most

7.

(a) for – but	(b) in– to
(c) a – the	(d) on – of
(e) a – an	(j) over – out
(g) for – of	(h) with – to

8.

(a) developed	development
(b) groups	group
(c) who	which
(d) make	made
(e) movement	movements
(f) which	who
(g) think	thought
(h) are	were
(i) for	of
(j) begin	began

[C] Omission

1.

(a) all of
(b) have some tea
(c) Being the eldest
(d) wasn't sufficient milk
(e) very little butter
(f) prepare some tea
(g) enjoyed the tea
(h) wasn't any tea

2. (a) Macbeth did not
(b) witches had promised
(c) had been proved
(d) mind began to
(e) It was only
(f) ambition started
(g) mind began to
(h) king was a

3. (a) This was the
(b) conspirators did what
(c) Brutus was the
(d) had done this
(e) honestly thought to
(f) He paid his
(g) life was gentle
(h) and say to

4. (a) idea struck them
(b) would disguise herself
(c) she told Celia
(d) would call herself
(e) Celia decided to
(f) were ready for

5. (a) bags were opened
(b) he walked/went to
(c) who was looking
(d) he knew well
(e) Selby gave him
(j) Everything went well
(g) always had, the
(h) would ask him

6. (a) of the system
(b) originated in London
(c) system of 68 men
(d) grew and poverty
(e) out of control
(f) formed the London
(g) force which Peel
(h) and was more
(i) disciplined than any
(j) ever had been

7. (a) beginnings in a
(b) Dion has risen
(c) superstardom of a
(d) Celine was the
(e) of a highly
(f) parents being musicians
(g) and at weekends
(h) entertained the local
(i) sang with her
(j) Celine had composed

8. (a) sport that can
(b) women and children
(c) in a gymnasium
(d) There is a
(e) of the end
(f) 3 m on the
(g) ball though it
(h) play with each other
(i) game by scoring
(j) ball into the

9. (a) decided to stop

(b) They no longer

(c) side has stopped

(d) There is no

(e) weapons though the

(f) weapons are still

(g) keep us safe

(h) scientists who made

(i) They do not

(j) save it for

10. (a) even in non

(b) particularly in managing

(c) and the systems

(d) along with the

(e) body that respond

(f) here as well.

[D] Jumbled Words

1. (a) It has a long forked tail on the back, a sheen and wings.

(b) It has a blue band with dark red beneath it at its throat

(c) Often swallows are seen skimming over a pond or an open space.

(d) Sometimes they may be found hunting insects down a busy city street.

2. (a) He feels that selling ice-cream is a tough business.

(b) There are just too many brands of ice-creams selling in the market now a days.

(c) A decade ago there were just two or three different varieties to choose from.

(d) Some people who try a new brand come back again and again for them.

(e) It's the King cones that sell like hot cakes.

(f) However there are some brands which are not favoured by customers.

3. (a) Concerted efforts should be made by all to curb environmental pollution.

(b) Burning of fossil fuels is to be discouraged.

(c) Polluting Industries must be shifted far off from the residential areas.

(d) More trees should be grown along the roads and in vacant fields.

(e) Though it will be a difficult task but certainly not impossible.

4. (a) What has being good to do with it?

(b) You will be rewarded for your good work.

(c) Show your medals to the Principal.

(d) How foolishly I have behaved !

5. (a) How can I yield to your desire ?

(b) How difficult the questions were !

(c) Newton had discovered the law of gravitation.

(d) Don't mix up with the bad boys.

6. (a) Autogenic training is based upon passive concentration.

(b) It has been given the status of a therapy by its disease alleviating capabilities.

(c) Autogenic training is effectively used for reducing stress.

(d) The exercise must be discontinued if some adverse side - effects are felt.

(e) It is not recommended for people with severe mental disorders.